Universal Digital Quantum Simulation with Trapped Ions

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Outline

- Analog & Digital simulation
- Trotter approximation
- Setup
- Toolbox
- Two spin models
- Three and six spin models
- Conclusions

Introduction

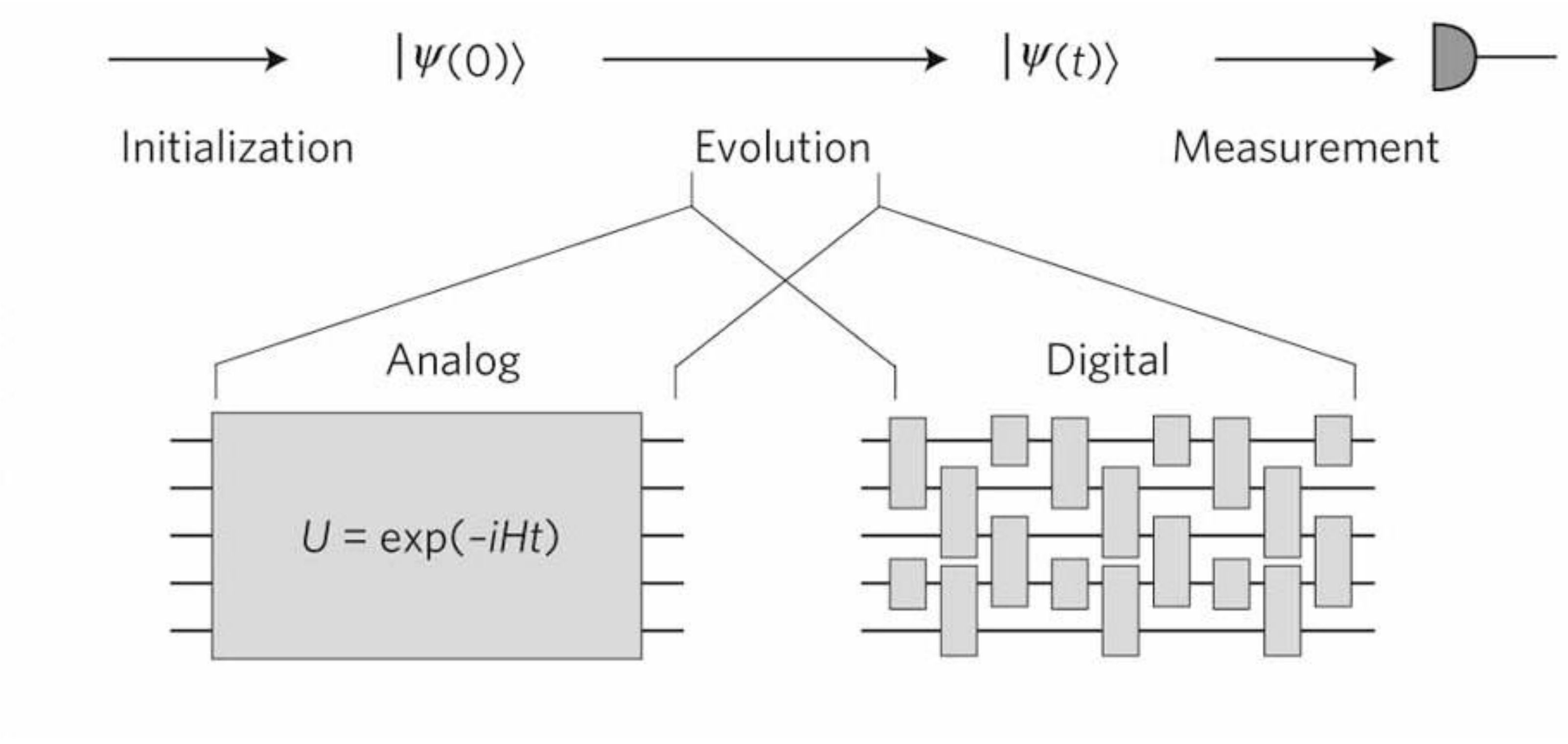
Many natural phenomena are described by quantum mechanics: to calculate properties of physical systems one can simulate quantum physics

With classical computers the problem increases exponentially with the number of particles involved

One can use quantum computers that can efficiently perform the simulation

Digital and Analog Quantum Simulation

The goal is to obtain $\psi(t) = e^{\frac{-i\hat{H}t}{\hbar}}\psi(0)$



The digital simulator operating under a universal set of unitaries forms a universal quantum simulator

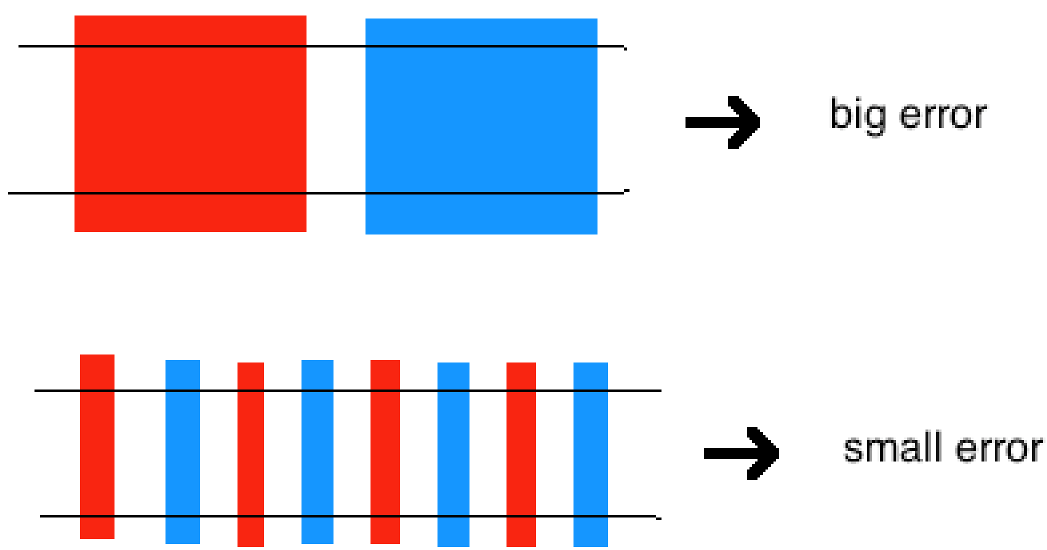
Trotter Approximation

In general the unitary U associted with the Hamiltonian H is

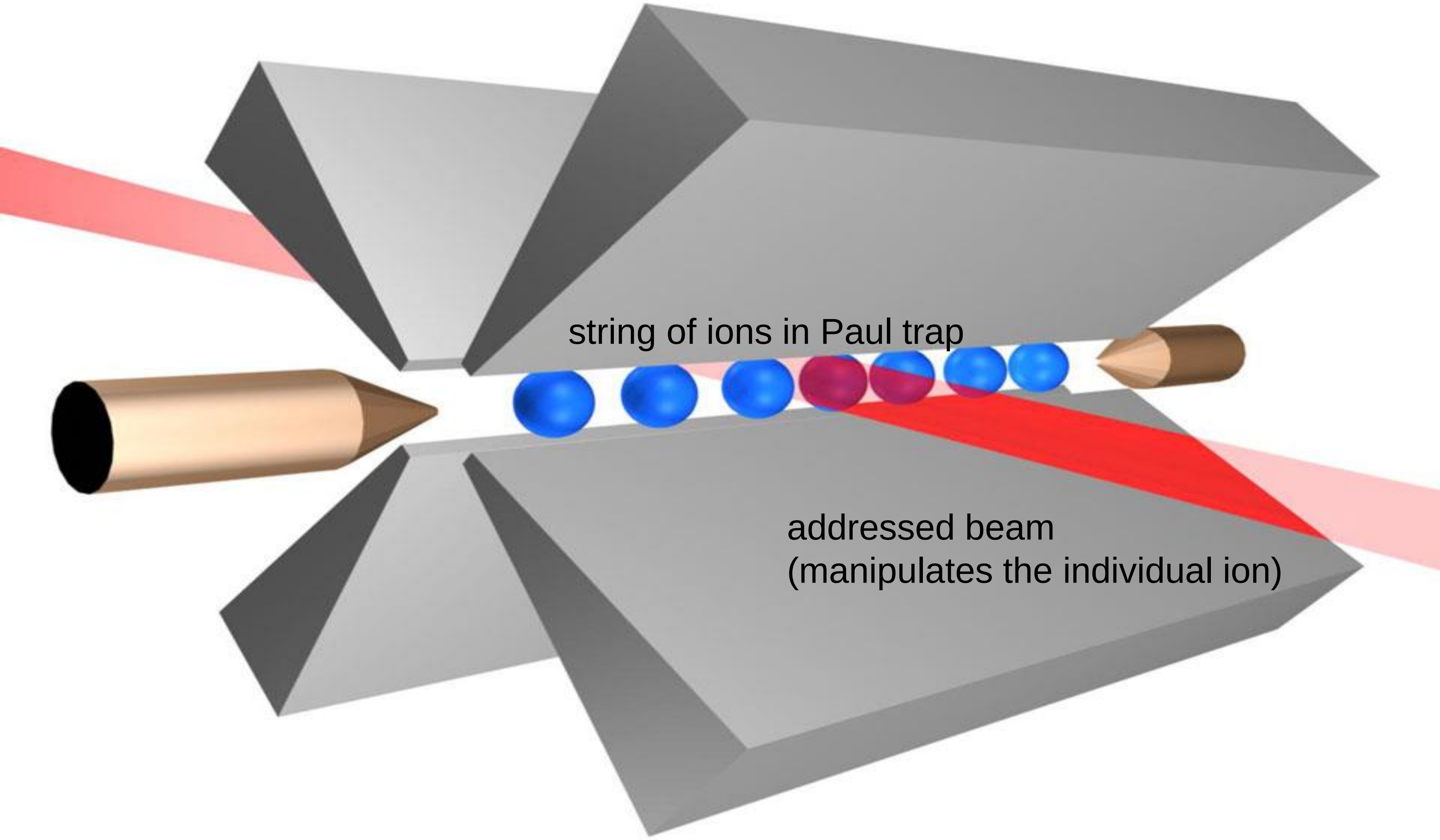
$U = e^{\frac{-i\sum_k \hat{H}_k t}{\hbar}} \neq \prod_k e^{\frac{-i\hat{H}_k t}{\hbar}}$ as the terms $\hat{H}_k$ do not always commute

However, according to the Trotter Approximation $U = \lim_{n\to\infty}\left(\prod_k e^{\frac{-i\hat{H}_k t}{\hbar n}}\right)^n$

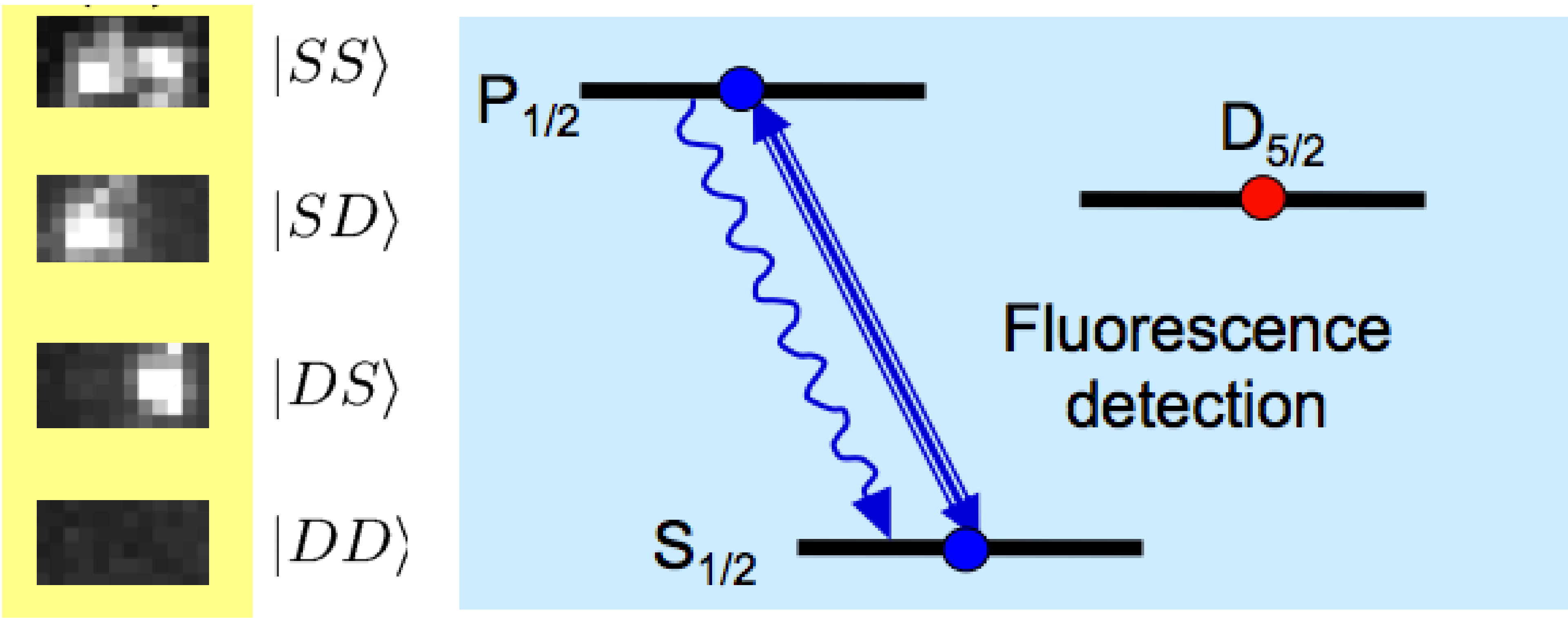
For finite n, the error can be made arbitrary small.



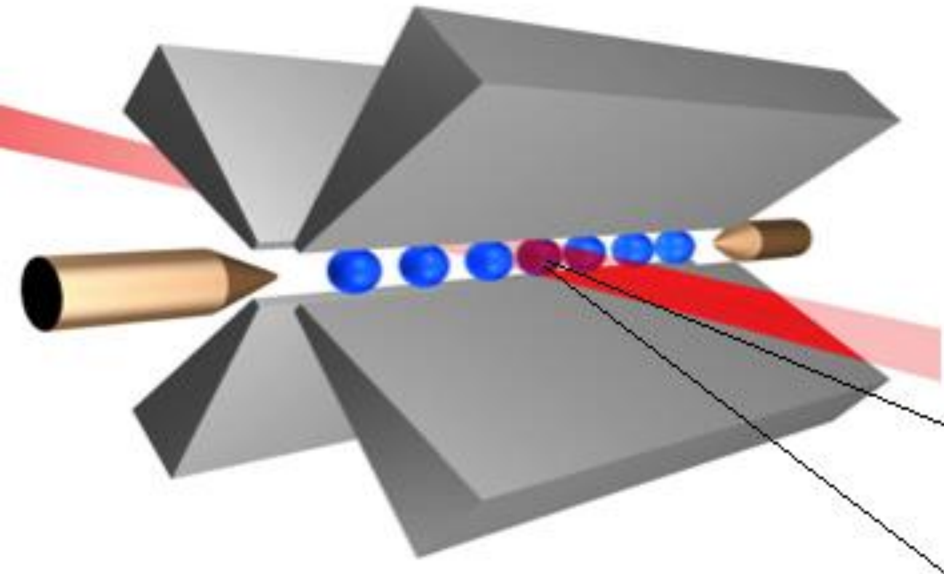
Ion Trap Setup

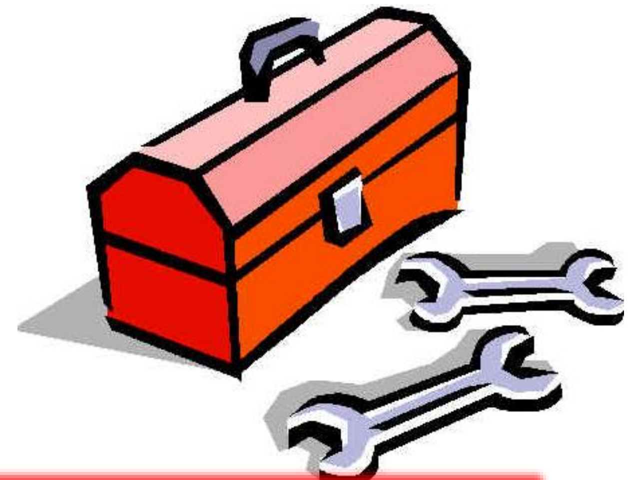


Zeeman states encode a qubit in each ion



Universal set of operators





Toolbox

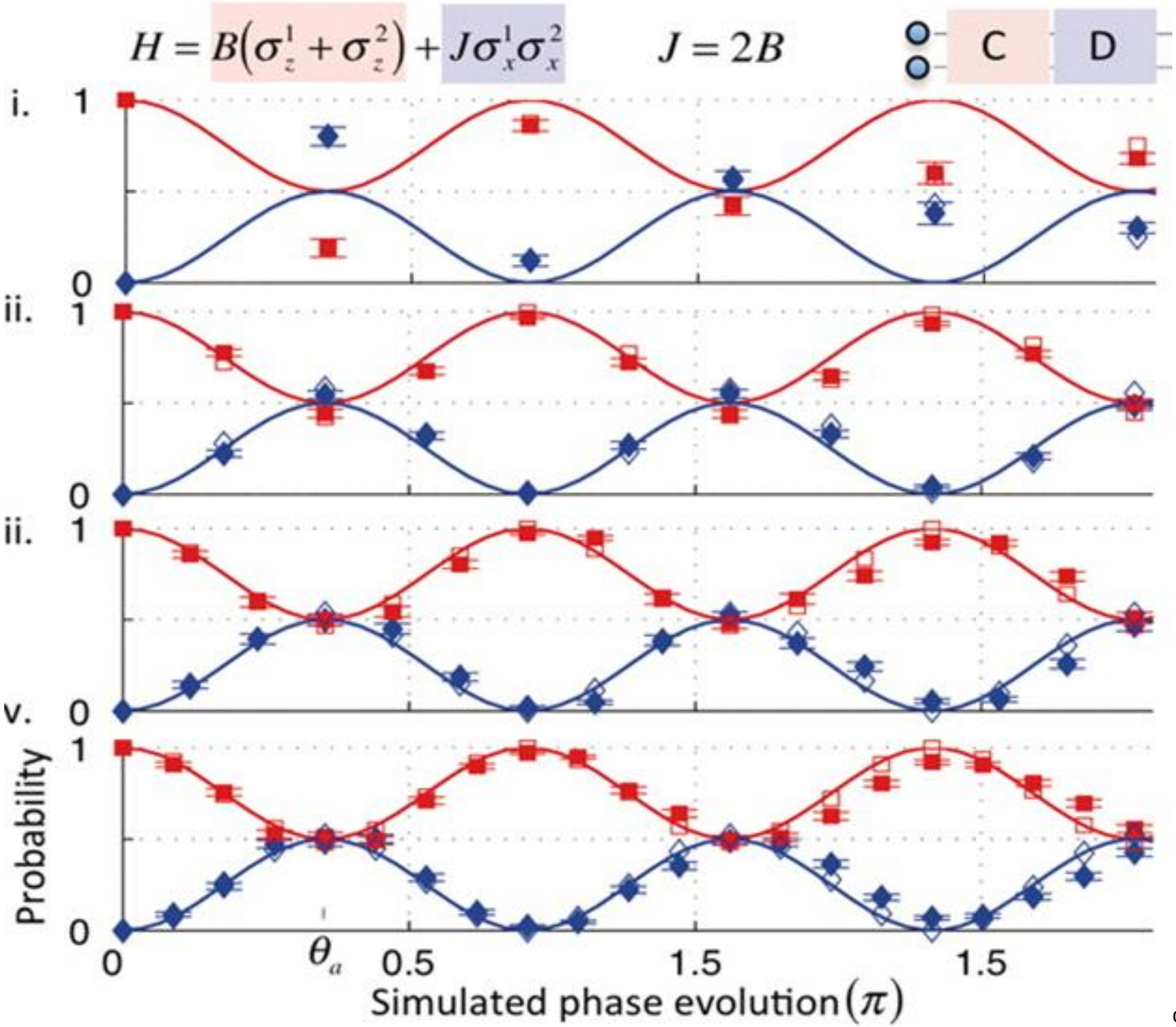
$$\begin{aligned}
O_1(\theta, i) &= \exp(-i\theta\sigma_z^i) \\
O_2(\theta) &= \exp(-i\theta \sum_i \sigma_z^i) \\
O_3(\theta, \phi) &= \exp(-i\theta \sum_i \sigma_\phi^i) \\
O_4(\theta, \phi) &= \exp(-i\theta \sum_{i<j} \sigma_\phi^i \sigma_\phi^j)
\end{aligned}$$

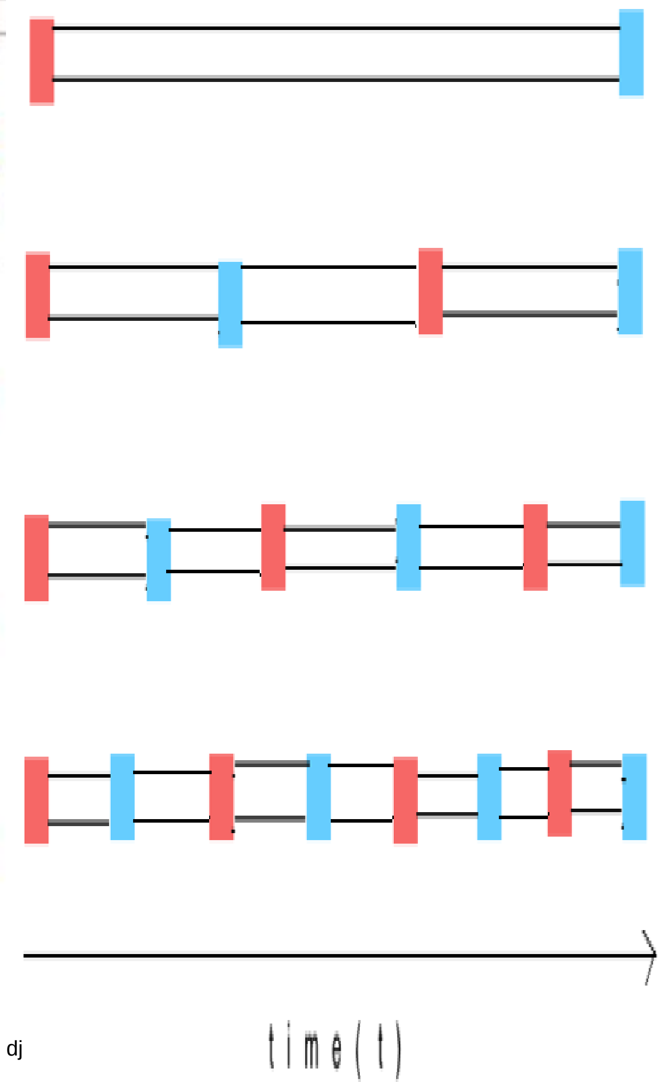
Where $\sigma_\phi = \cos\phi\,\sigma_x + \sin\phi\,\sigma_y$

$\theta = Et/\hbar$

Two spin Ising chain

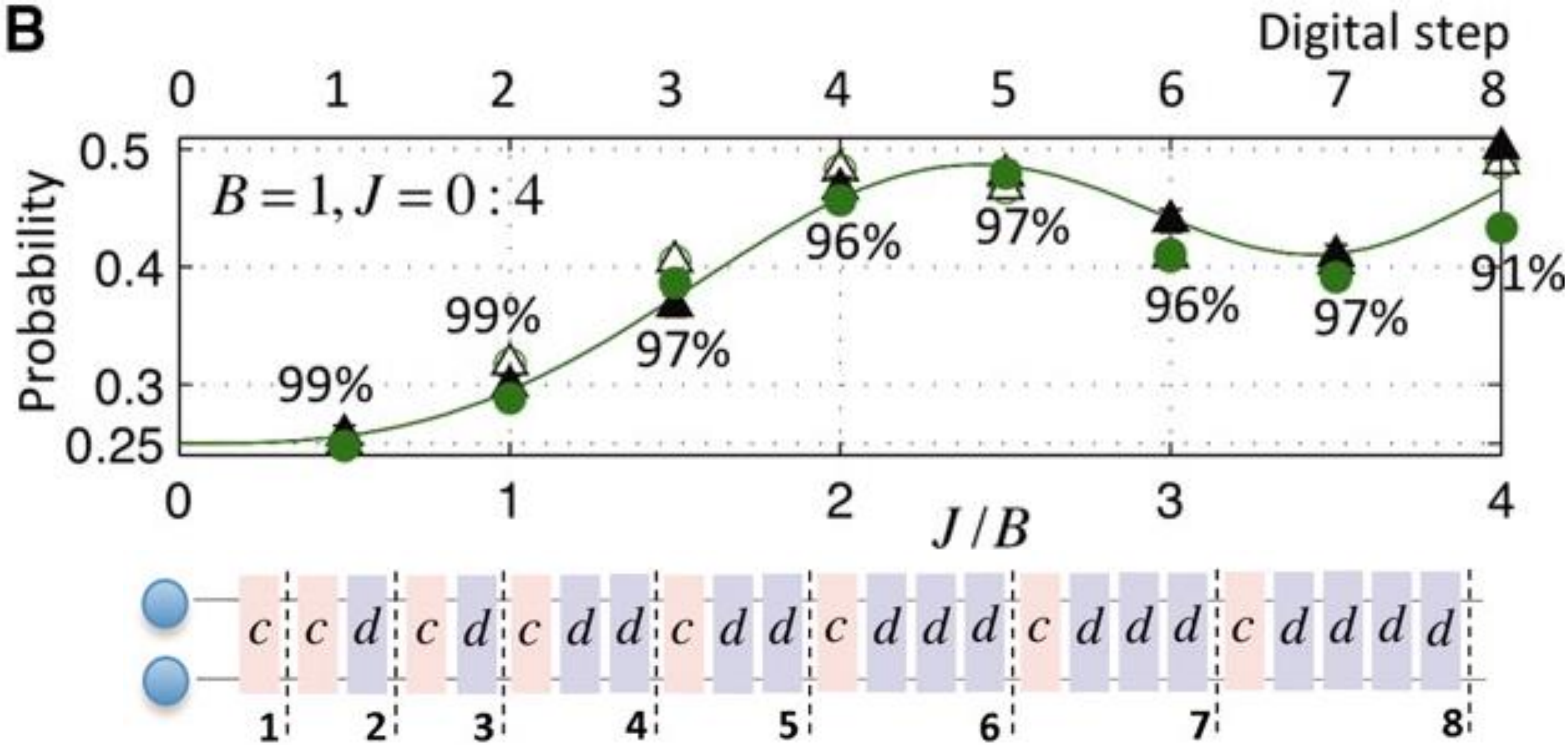
$|\uparrow\uparrow\rangle$





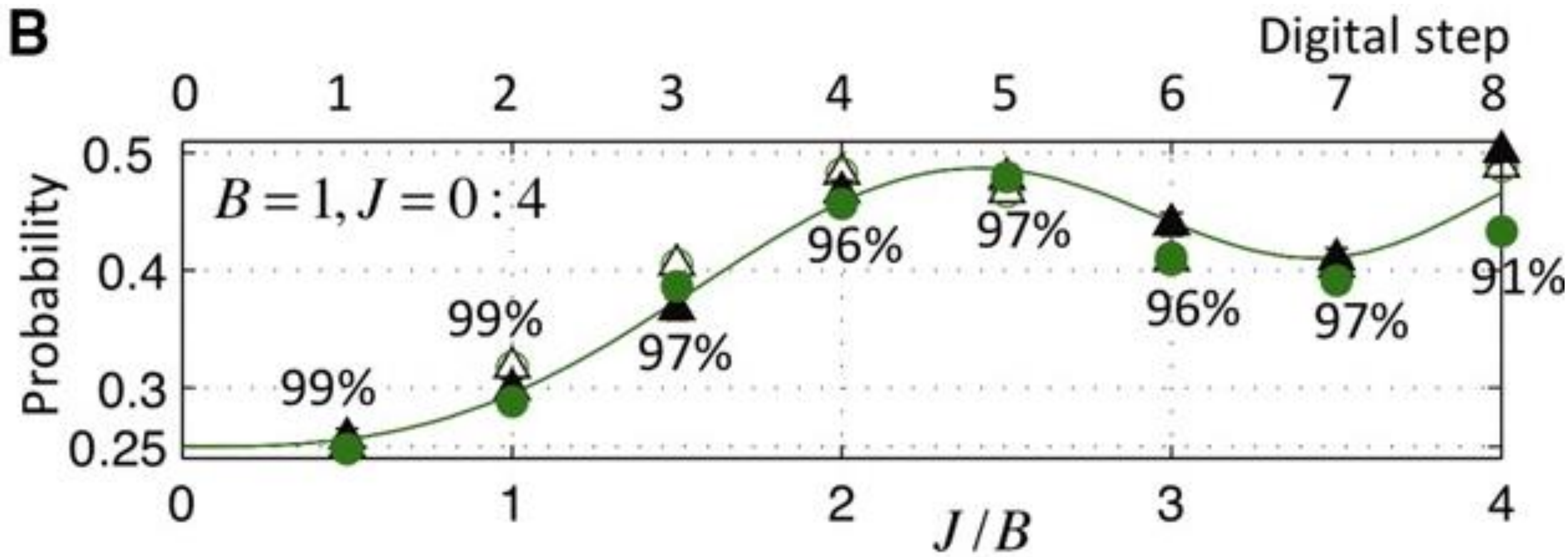
Two spin Ising chain

$|\uparrow\uparrow\rangle$



Two spin Ising chain

$$|\uparrow\uparrow\rangle$$



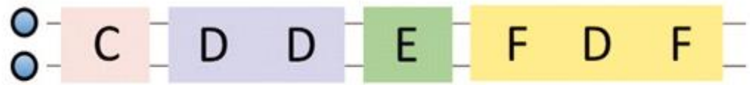
$$\frac{1}{2}(|\rightarrow\rightarrow\rangle_x + |\leftarrow\leftarrow\rangle_x + |\rightarrow\leftarrow\rangle_x + |\leftarrow\rightarrow\rangle_x) \rightarrow \frac{1}{\sqrt{2}}(|\rightarrow\rightarrow\rangle_x + |\leftarrow\leftarrow\rangle_x)$$

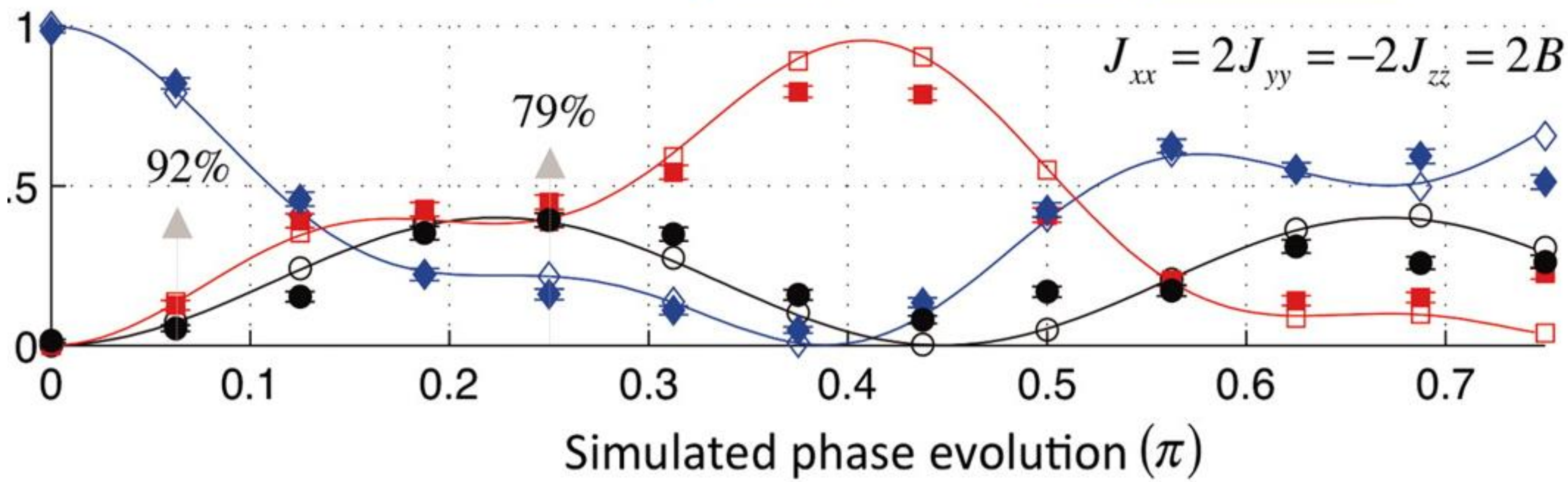
The XYZ Model

XYZ

$$H = B(\sigma_z^1 + \sigma_z^2) + J_{xx}\sigma_x^1\sigma_x^2 + J_{yy}\sigma_y^1\sigma_y^2 + J_{zz}\sigma_z^1\sigma_z^2$$

$|\rightarrow\leftarrow\rangle_x$

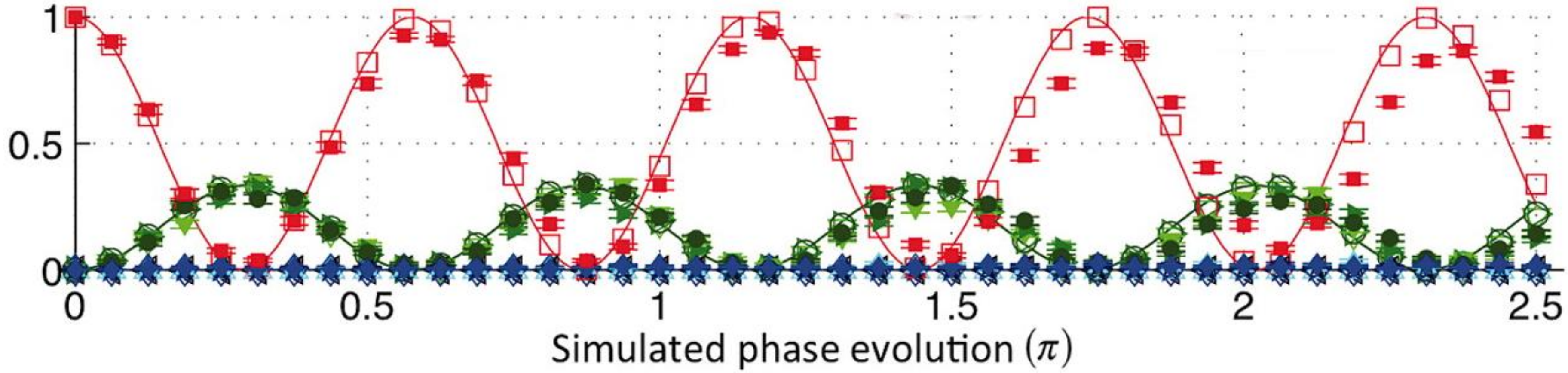




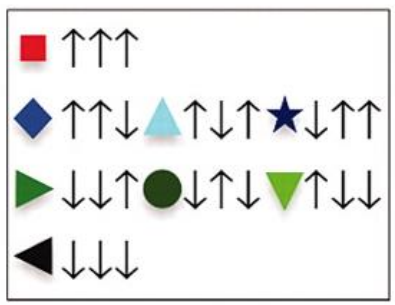
(◆ $\rightarrow\leftarrow_x$ ■ $\leftarrow\rightarrow_x$ ● $\leftarrow\leftarrow_x$ or $\rightarrow\rightarrow_x$)

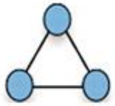
$$C = O_2(\frac{\pi}{16}), D = O_4(\frac{\pi}{16}, 0), E = O_4(\frac{\pi}{16}, \frac{\pi}{2}), F = O_3(\frac{\pi}{4}, 0)$$

Long-range 3 spin Ising system



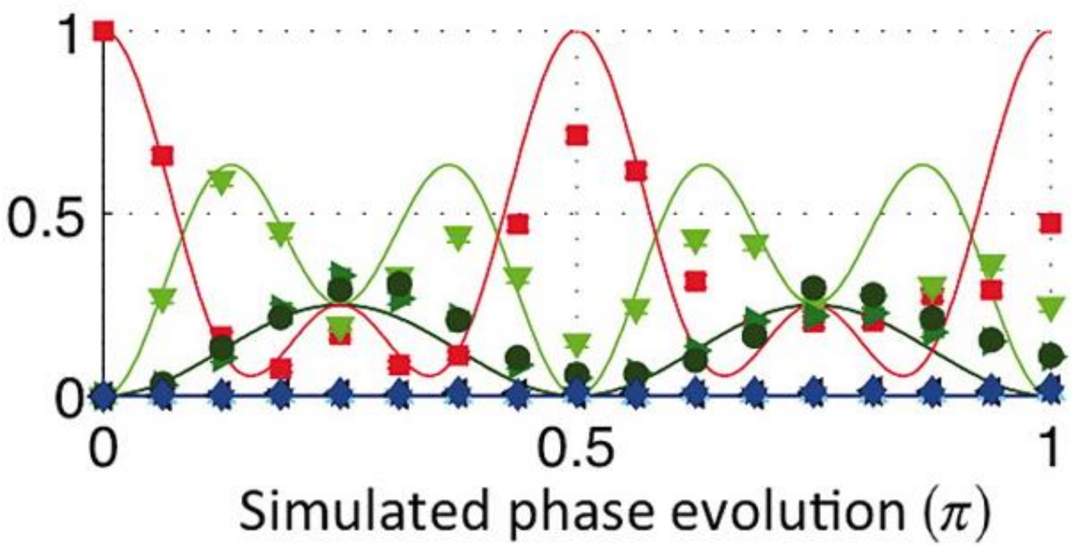
$$H = J\sum_{i<j}\sigma_x^i\sigma_x^j + B\sum_k \sigma_z^k$$



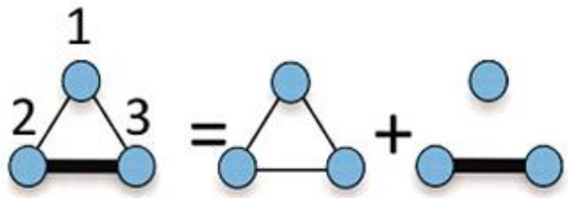


$$J = 2B$$

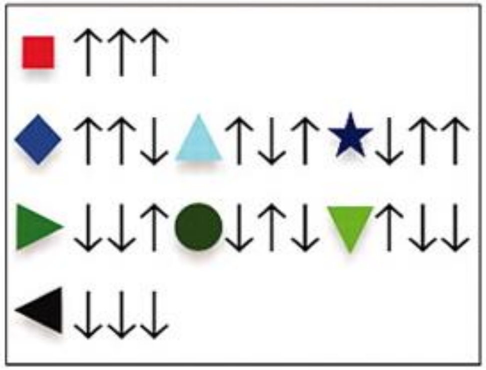
Inhomogeneous distribution of spin-spin couplings



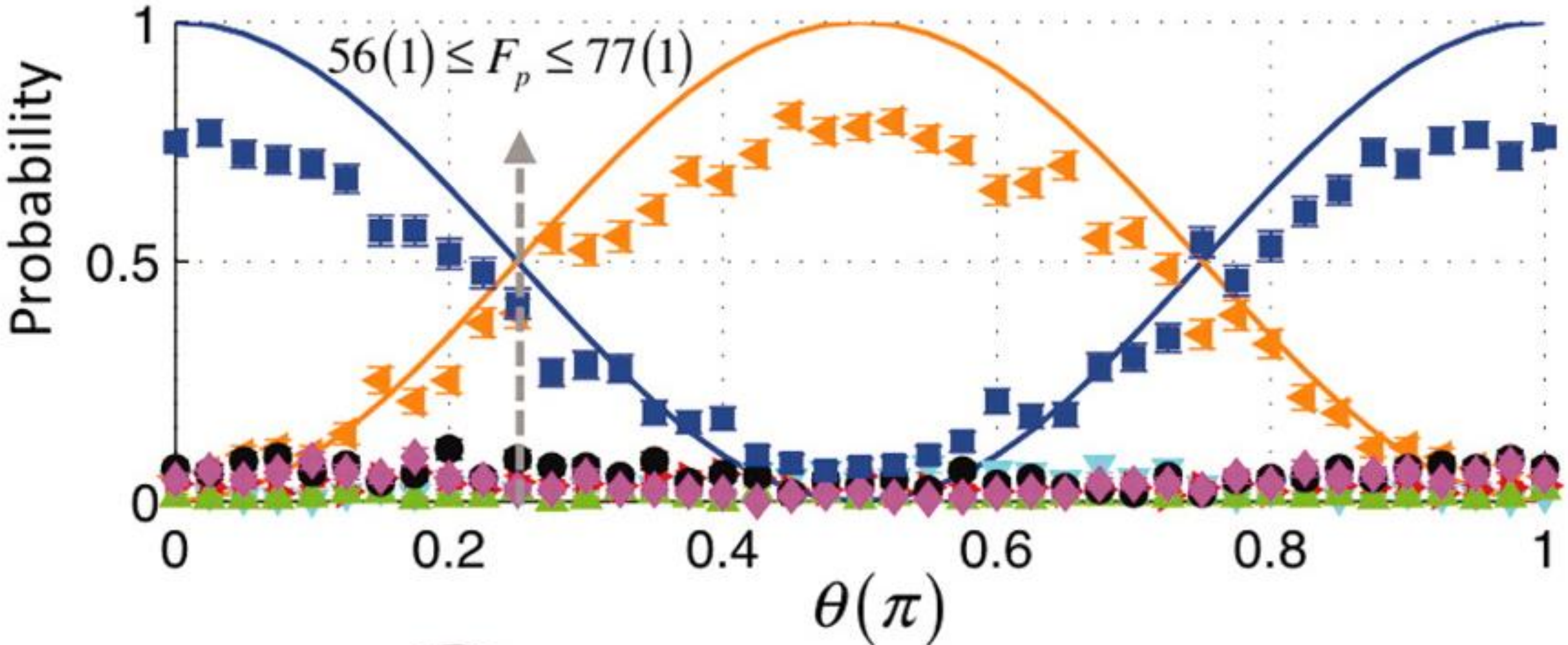
$$H = \sum_{i<j} J_{ij}\sigma_x^i\sigma_x^j$$



$$J_{23} = 3J_{12} = 3J_{13}$$



Six spin six-body interaction



$$H = \sigma_y^1 \sigma_x^2 \sigma_x^3 \sigma_x^4 \sigma_x^5 \sigma_x^6$$

filled shapes: data (■P_0 ◆P_1 ●P_2 ▲P_3 ►P_4 ▼P_5 ◄P_6, where P_i is the total probability of finding *i* spins pointing down).

Conclusions

- The Trotter approximation allows for accurate digital simulation
- Trapped ions serve as a strong proof of concept universal quantum computer
- Hamiltonians of up to six interacting particles are simulated to reasonable accuracy