ETH zürich

Dissipation in Transmon

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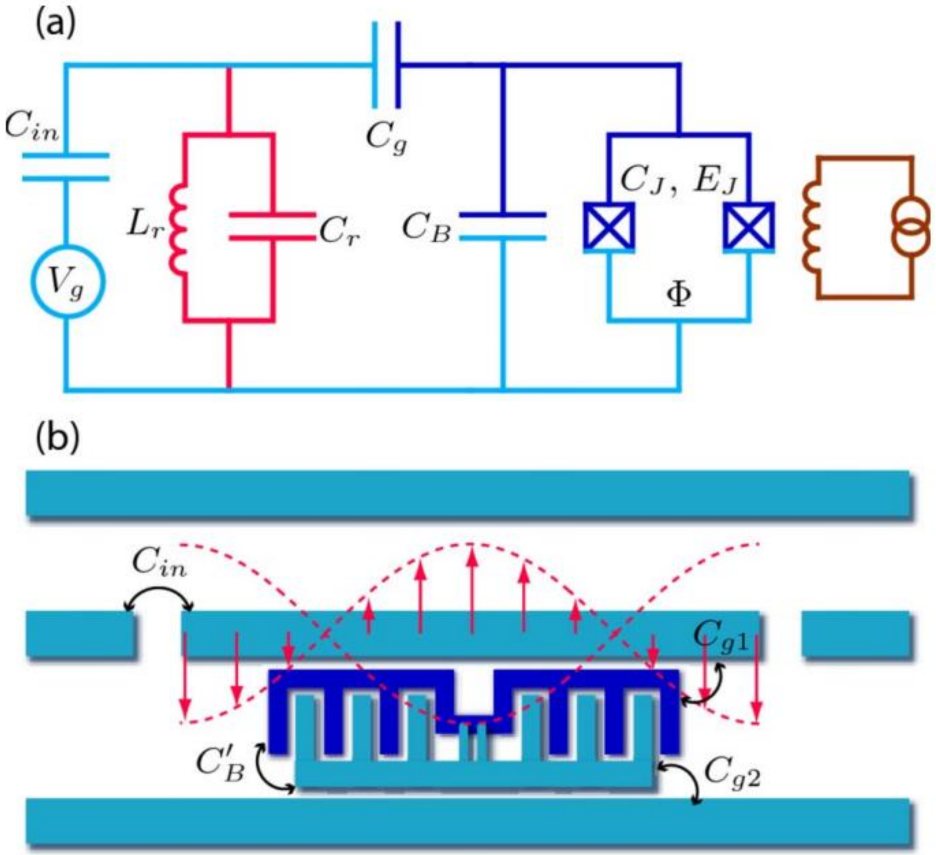
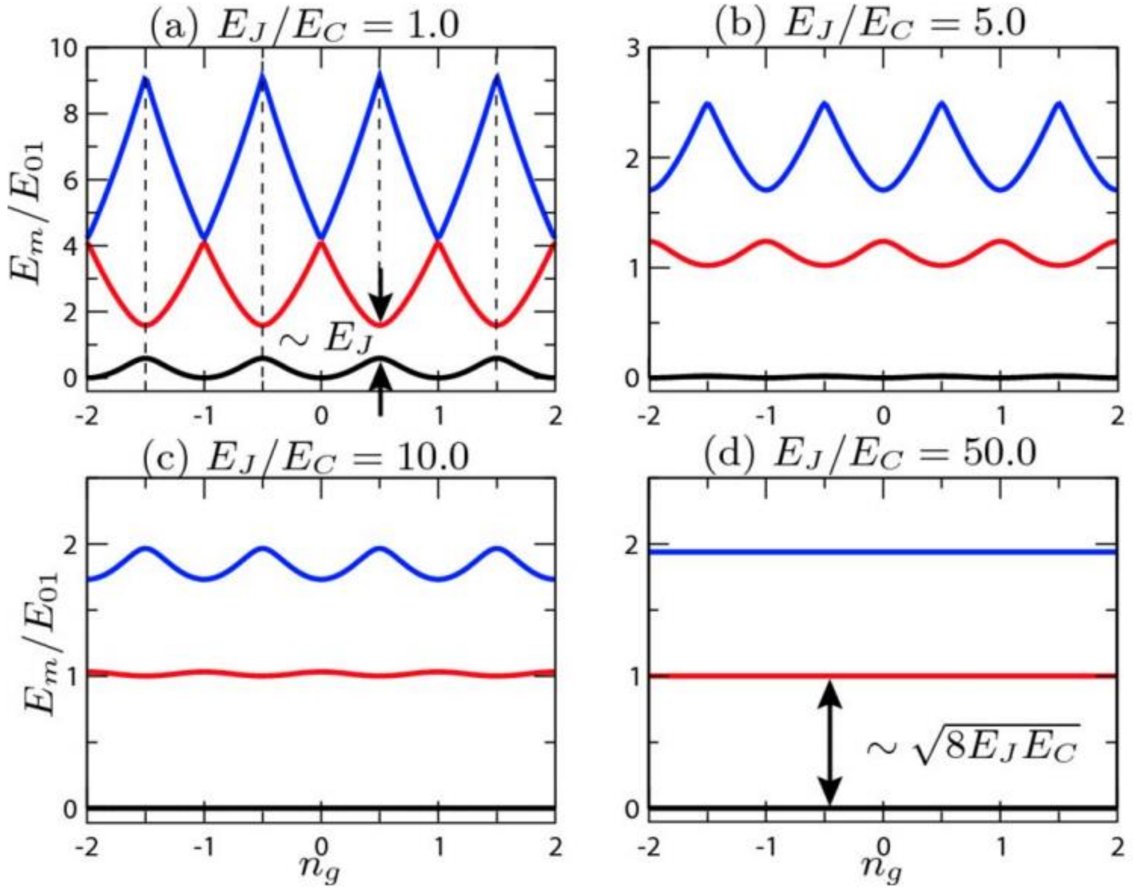
Highlight



Koch et al. - 2007 - Charge-insensitive qubit design derived from the Cooper pair box

- The large E_J/E_C ratio and the low energy dispersion contribute to Transmon's most significant advantage---**insensitivity to charge noise**.
- $E_C = e^2/2(C_g + C_B + C_J)$
- This presentation will concentrate on the dissipation analysis performed by Koch et al., that is, noise performance. Introduce the basic ideas of dissipation and present their results.

Highlight



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- Transmon could be proved to have identical Hamiltonian as Cooper Pair Box. The distinguishing difference is the shunt capacitor which significantly increase the ratio E_J/E_C.

Outline

- General idea of dissipation and decoherence
- Relaxation time---Amplitude damping channel
- Dephasing time---Phase damping channel

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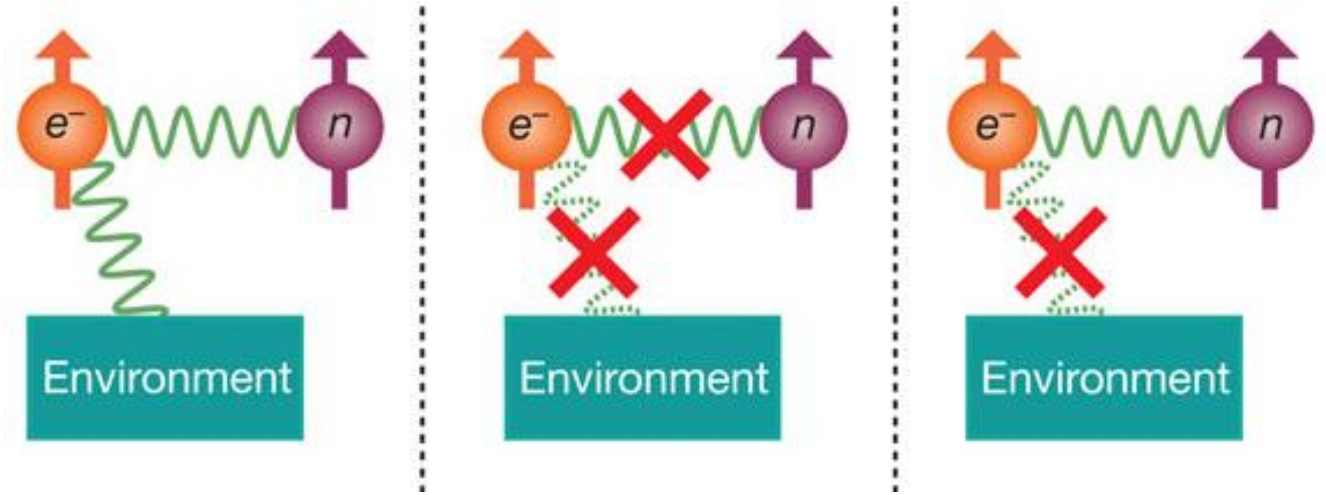
General Ideas

- Loss of entanglement or the leak of quantum information.
- From quantum realm to classical realm, like a projection.
- We will include the state of the environment for convenience.

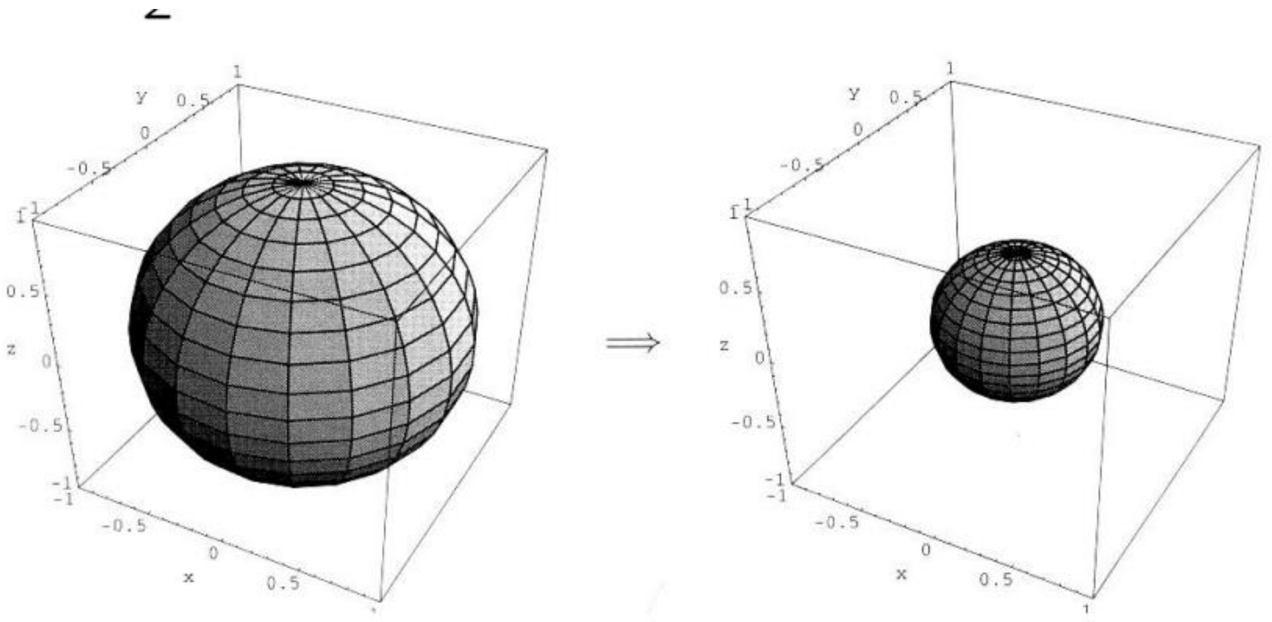


T Sar *et al*. *Nature* **484**, 82-86 (2012) doi:10.1038/nature10900

General Ideas

- **Depolarization channel**

$$|\psi\rangle_A \otimes |0\rangle_E \rightarrow \sqrt{1-p}|\psi\rangle_A \otimes |0\rangle_E + \sqrt{\frac{p}{3}}\sum_{i=1}^{3} \sigma_i\,|\psi\rangle_A \otimes |i\rangle_E$$



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Depolarization channel

- Bit flip error

$$|\psi\rangle \rightarrow \sigma_x|\psi\rangle$$

- Phase flip error

$$|\psi\rangle \rightarrow \sigma_z|\psi\rangle$$

- Both

$$|\psi\rangle \rightarrow \sigma_y|\psi\rangle$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

General Ideas

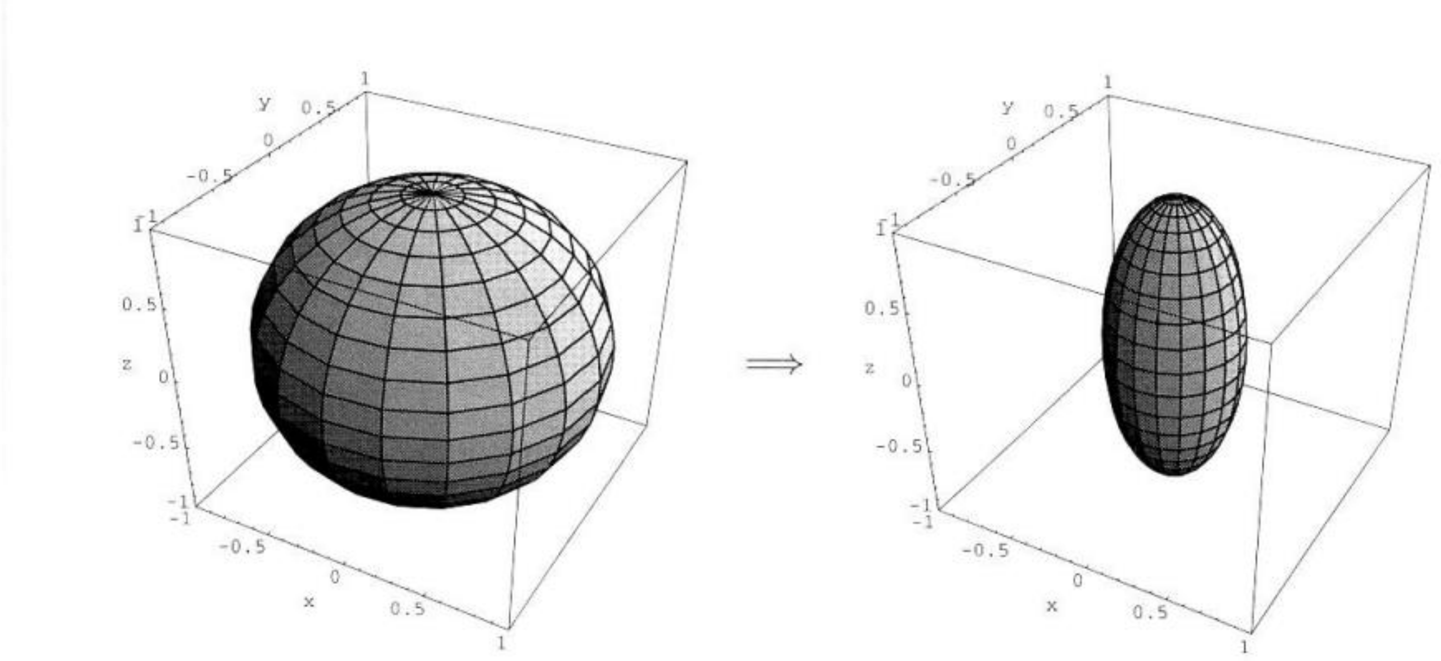
- Bloch sphere representation of Density matrix

$$\rho = \frac{1}{2}(I + \boldsymbol{\sigma} \cdot \boldsymbol{r})$$

General Ideas

- **Dephasing channel**

$$|0\rangle_A \otimes |0\rangle_E \to \sqrt{1-p}|0\rangle_A \otimes |0\rangle_E + \sqrt{p}|0\rangle_A \otimes |1\rangle_E$$



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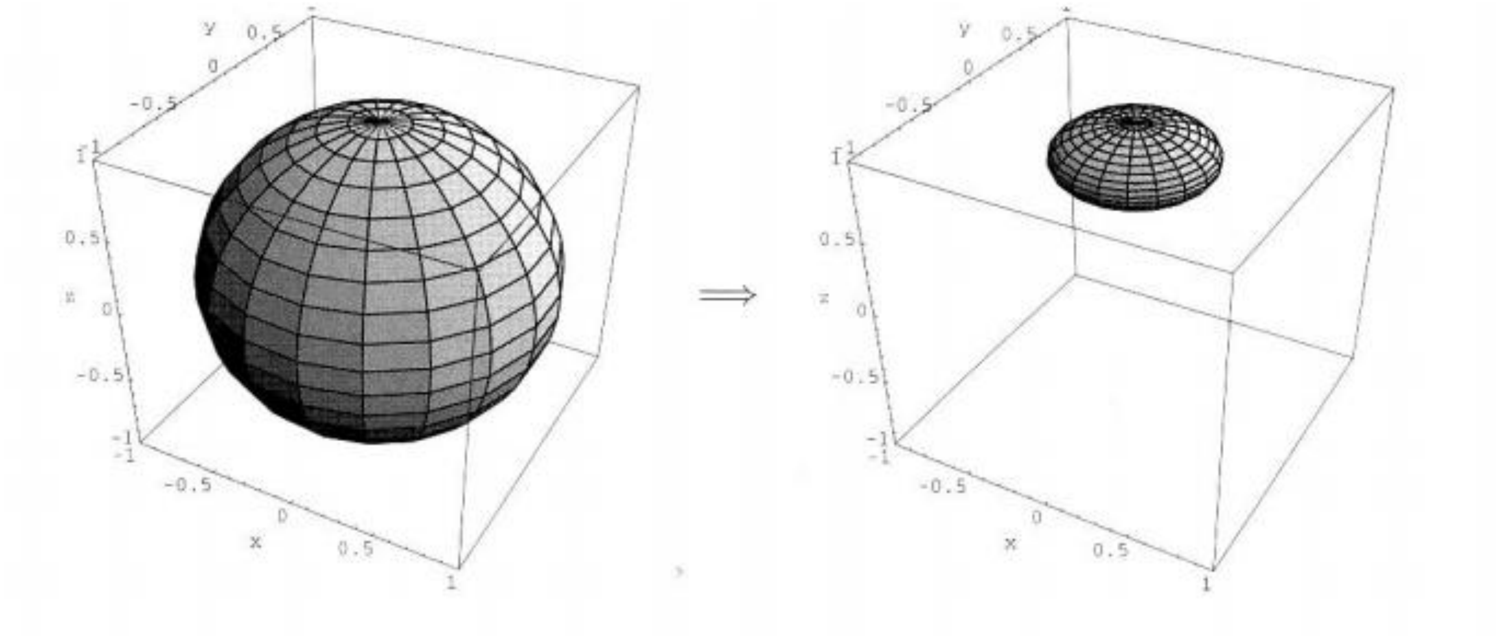
General Ideas

- **Relaxation channel**

$$|1\rangle_A \otimes |0\rangle_E \rightarrow \sqrt{1-p}|1\rangle_A \otimes |0\rangle_E + \sqrt{p}|0\rangle_A \otimes |1\rangle_E$$



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Relaxation time---Amplitude damping channel

- Spontaneous Emission
- Purcell Effect
- Dielectric losses
- Quasiparticle tunneling
- Flux coupling
- General discussion: other sources
- Channel addition rule

$$\Gamma = \Gamma_\alpha + \Gamma_\beta + \cdots$$
$$\frac{1}{\mathrm{T}} = \frac{1}{T_\alpha} + \frac{1}{T_\beta} + \cdots$$

Relaxation time---Amplitude damping channel

- Spontaneous Emission
  - Spontaneous emission power

$$P = \frac{1}{4\pi\varepsilon_0}\frac{d^2\omega^4}{3c^3}$$

  - The oscillation is caused by cooper pairs, whose traveling (tunneling) distance is 15um
  - In this case we have

$$T_1^{se} = \frac{\hbar\omega_1}{P} = \frac{12\pi\varepsilon_0\hbar c^3}{d^2\omega_{01}^3} \sim 0.3ms$$

Relaxation time---Amplitude damping channel

- Purcell Effect
  - When the system is put into a resonator, a cavity for instance, its Spontaneous Emission rate would be altered.
  - The Purcell effect is a modification as the ratio of cavity modes density of state to free space modes density of state

$$F = \frac{\rho_c}{\rho_f} = \frac{1}{\Delta\nu V} / \frac{8\pi n^3 \nu^2}{c^3}$$

Relaxation time---Amplitude damping channel

- Purcell Effect
  - We define the transition rate

$$\gamma_k^{f,i} = \frac{2\pi}{\hbar} p(\omega_k) \left| \langle 1_k, f \left| \hbar \sum_k \lambda_k [\hat{b}_k^+ \hat{a} + \hat{a}^+ \hat{b}_k] \right| 0, i \rangle \right|^2$$

  - In the dispersive limit the transmon states acquire only a small photonic component, it means we could treat it as a perturbation
  - After first order perturbation calculation, we get

$$\gamma_k^{f,i} = \frac{2\pi}{\hbar} p(\omega_k) \lambda_k^2 \frac{g_{f,i}^2}{(\omega_{f,i} - \omega_r)^2}$$

  - We thus get modified $T_1 \sim 16\mu s$

Relaxation time---Amplitude damping channel

- **Dielectric losses**

- In insulating materials, especially amorphous SiO_2, it would affect the electric fields associated with the qubits and cause energy relaxation.

- In particular, in the transmon design the shunting capacitance C_B offers the possibility to accumulate a large percentage of the electric fields in a well controlled spatial region with favorable substrates,

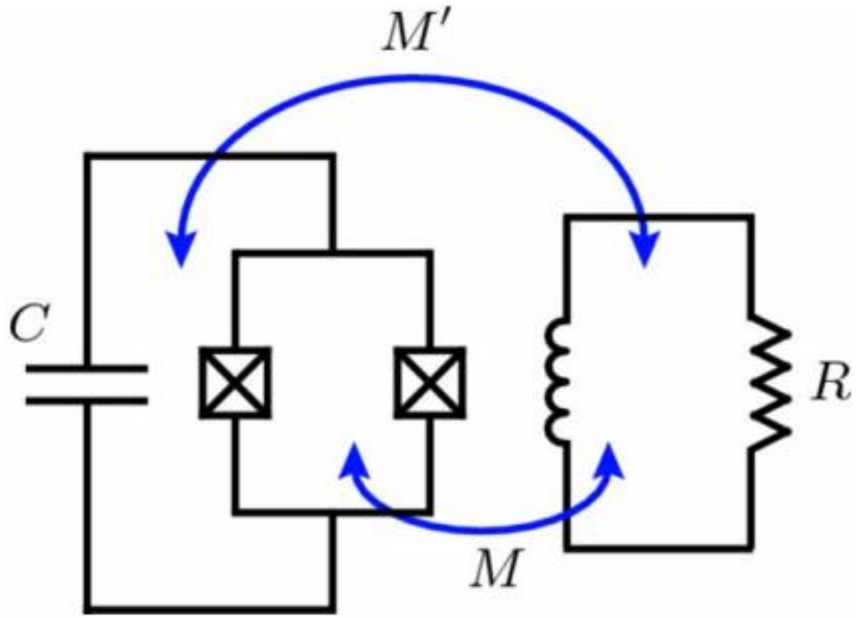
Relaxation time---Amplitude damping channel

- Quasiparticle tunneling
  - Following the arguments by Lutchyn *et al.*

$$N_{qp} = 1 + \frac{3\sqrt{2}}{2} N_e \frac{\sqrt{\Delta k_B T}}{E_F} e^{-\Delta/k_B T}$$

  - valid for temperatures small compared to the superconducting gap Δ
  - $T_1 \sim 1s$ in this channel when temperature is below $100mK$

Relaxation time---Amplitude damping channel



- There is an intentional coupling between the SQUID loop and the flux bias (allowing for the E_J tuning) through a mutual inductance *M*
- In addition, the entire transmon circuit couples to the flux bias via a mutual inductance *M*'
- We choose to assume that the applied flux could be decomposed into external flux and a small noise term, $\Phi = \Phi_e + \Phi_n, \Phi_n \ll \Phi_e$

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Relaxation time---Amplitude damping channel

- Flux coupling
  - Then we Taylor expand the Josephson Hamiltonian

$$H_J \rightarrow H_J + \Phi_n A, A = \frac{\partial H_J}{\partial \Phi}\Big|_{\Phi_e}$$

  - In this way, we could write the dissipation rate (just transition rate)

$$\Gamma_1 = \frac{1}{\hbar^2}|\langle 1|A|0\rangle|^2 S_{\Phi_n}(\omega) = \frac{1}{\hbar^2}|\langle 1|A|0\rangle|^2 M^2 S_{I_n}(\omega)$$

  - T_1: $20ms \sim 1s$
  - However, new research found that there is capacitive coupling that contribute much more to the decoherence time μs

Relaxation time---Amplitude damping channel

- Other sources
  - Coupling to spurious resonator modes
  - Pinning and unpinning of vortices
  - Bulk piezoelectricity

Dephasing time---Phase damping channel

- Analytical discussion
- Charge noise & Flux noise
- Critical current noise
- Quasiparticle tunneling
- E_C noise

Dephasing time---Phase damping channel

- The energy shift of the eigenstates would cause a change of transition frequency.
- Then it would lead to a decay in the off diagonal elements
- In formal theory,

$$H_q = \frac{1}{2}\sum_{u=x,y,z} h_u(\{\lambda_i\})\hat{\sigma}_u, \qquad \lambda_i = \lambda_i^0 + \delta\lambda_i$$

- Then do Taylor Expansion, calculate the noise power by autocorrelation. We focus on the dephasing aspect, low-frequency noise in the σ_z component.

$$H_q = \frac{\hbar\omega_{01}}{2}\sigma_z + \frac{1}{2}\sum_j \frac{\partial h_z(\{\lambda_i\})}{\partial\lambda_j}\delta\lambda_j\sigma_z + O(\delta\lambda^2)$$

Dephasing time---Phase damping channel

- Then do Taylor Expansion, calculate the noise power by autocorrelation, we get the law of off diagonal elements

$$\rho_{01}(t) = e^{i\omega_{01}t} \exp\left(-\frac{1}{2}\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} S_v(\omega) \frac{\sin^2(\omega t/2)}{(\omega/2)^2}\right)$$

- When $t \gg t_c$,

$$\rho_{01}(t) = e^{i\omega_{01}t} \exp\left(-\frac{1}{2}|t| S_v(\omega = 0)\right)$$

- Dephasing time $T_2 \sim 2/S_v(\omega = 0)$, Lorentzian lineshape

Dephasing time---Phase damping channel

- Charge noise
  - Small fluctuations regime

$$T_2 \sim \frac{\hbar}{A} \left| \frac{\partial E}{\partial n_g} \right|^{-1} \sim 8s$$

  - Slow charge fluctuations with large amplitudes

$$H_q = \frac{1}{2}\left(\hbar\omega_{01} + \frac{\epsilon_1}{2}\cos\left[2\pi n_g + 2\pi\delta n_g(t)\right]\right)\sigma_z$$

$$T_2 \sim \frac{4\hbar}{e^2\pi|\epsilon_1|} \sim 0.4ms, \qquad Transmon\ in\ worst\ case$$

$$T_2 \sim \left| \frac{\pi^2 A^2}{\hbar^2} \frac{\partial^2 E}{\partial n_g^2} \right|^{-1} = \frac{\hbar^2}{\pi^2 A^2} \frac{E_J}{64 E_C} \sim 1\mu s, \qquad Cooper\ Pair\ Box\ at\ sweet\ point$$

Dephasing time---Phase damping channel

- Flux noise
  - Noise in the externally applied flux translates into fluctuations of the effective Josephson coupling energy E_J.

$$T_2 \sim \left| \frac{\pi^2 A^2}{\hbar^2} \frac{\partial^2 E}{\partial \Phi^2} \right|^{-1} \sim 3.6ms, \qquad at\ integer\ flux\ quanta, sweet\ point$$

Dephasing time---Phase damping channel

- Critical current noise
  - Critical current noise is likely to be the limiting dephasing mechanism. Such rearrangements in the junction directly influence the critical current and hence the Josephson coupling energy $E_J = I_c\hbar/2e$.

$$T_2 \sim \frac{\hbar}{A}\left|\frac{\partial E}{\partial I_c}\right|^{-1} \sim 35\mu s$$

  - An improvement of a factor of 2

Dephasing time---Phase damping channel

- Quasiparticle tunneling
  - In CPB case, $\Gamma_2^{qp} = \Gamma^{qp} N_{qp}$, consider a jump from sweet point to half charge point
  - But in transmon regime, due to large ratio of E_J/E_C, the charge dispersion is exponentially flat so that transition frequency variations due to a single charge are minimal.
- E_C noise
  - A loophole?
  - Fluctuations in the effective capacitances of the circuit.
  - No current reports of this noise source.

Dephasing time---Phase damping channel

Noise source	$1/f$ amplitude	Transmon $E_J/E_C=85$ T_2 (ns)	CPB $E_J/E_C=1$ T_2 (ns)
Charge	$A=10^{-4}-10^{-3}e$ [51]	400 000	**1 000**[a]
Flux	$A=10^{-6}-10^{-5}\Phi_0$ [52,54]	3 600 000[a]	1 000 000[a]
Critical current	$A=10^{-7}-10^{-6}I_0$ [53]	**35 000**	17 000

[a]These values are evaluated at a sweet spot (i.e., second-order noise).

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References

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Thank You!