Dissipation in Transmon

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The large $E_J/E_C$ ratio and the low energy dispersion contribute to Transmon’s most significant advantage—insensitivity to charge noise.

$E_C = e^2/2(C_g + C_B + C_J)$

This presentation will concentrate on the dissipation analysis performed by Koch et al., that is, noise performance. Introduce the basic ideas of dissipation and present their results.
Transmon could be proved to have identical Hamiltonian as Cooper Pair Box. The distinguishing difference is the shunt capacitor which significantly increase the ratio $E_J/E_C$. 

Koch et al. - 2007 - Charge-insensitive qubit design derived from the Cooper pair box
Outline

- General idea of dissipation and decoherence
- Relaxation time---Amplitude damping channel
- Dephasing time---Phase damping channel

Koch et al. - 2007 - Charge-insensitive qubit design derived from the Cooper pair box
General Ideas

- Loss of entanglement or the leak of quantum information.
- From quantum realm to classical realm, like a projection.
- We will include the state of the environment for convenience.

T Sar et al. Nature 484, 82-86 (2012) doi:10.1038/nature10900
General Ideas

- Depolarization channel

\[ |\psi\rangle_A \otimes |0\rangle_E \rightarrow \sqrt{1-p} |\psi\rangle_A \otimes |0\rangle_E + \frac{\sqrt{p}}{3} \sum_{i=1}^{3} \sigma_i |\psi\rangle_A \otimes |i\rangle_E \]
Depolarization channel

- Bit flip error
  \[ |\psi\rangle \rightarrow \sigma_x |\psi\rangle \]

- Phase flip error
  \[ |\psi\rangle \rightarrow \sigma_z |\psi\rangle \]

- Both
  \[ |\psi\rangle \rightarrow \sigma_y |\psi\rangle \]

\[
I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]
General Ideas

- Bloch sphere representation of Density matrix

\[ \rho = \frac{1}{2} (I + \sigma \cdot r) \]
General Ideas

- Dephasing channel

\[ |0\rangle_A \otimes |0\rangle_E \rightarrow \sqrt{1-p} |0\rangle_A \otimes |0\rangle_E + \sqrt{p} |0\rangle_A \otimes |1\rangle_E \]
General Ideas

- Relaxation channel

\[ |1\rangle_A \otimes |0\rangle_E \rightarrow \sqrt{1-p}|1\rangle_A \otimes |0\rangle_E + \sqrt{p}|0\rangle_A \otimes |1\rangle_E \]
Relaxation time---Amplitude damping channel

- Spontaneous Emission
- Purcell Effect
- Dielectric losses
- Quasiparticle tunneling
- Flux coupling
- General discussion: other sources
- Channel addition rule

\[ \Gamma = \Gamma_\alpha + \Gamma_\beta + \cdots \]
\[ \frac{1}{T} = \frac{1}{T_\alpha} + \frac{1}{T_\beta} + \cdots \]
Spontaneous Emission

- Spontaneous emission power

\[ P = \frac{1}{4\pi \varepsilon_0} \frac{d^2 \omega^4}{3c^3} \]

- The oscillation is caused by cooper pairs, whose traveling (tunneling) distance is 15um

- In this case we have

\[ T_{1}^{se} = \frac{\hbar \omega_1}{P} = \frac{12\pi \varepsilon_0 \hbar c^3}{d^2 \omega_{01}^3} \approx 0.3ms \]
Purcell Effect

When the system is put into a resonator, a cavity for instance, its Spontaneous Emission rate would be altered.

The Purcell effect is a modification as the ratio of cavity modes density of state to free space modes density of state

\[
F = \frac{\rho_c}{\rho_f} = \frac{1}{\Delta \nu V} \frac{8 \pi n^3 v^2}{c^3}
\]


Relaxation time---Amplitude damping channel

- **Purcell Effect**
  - We define the transition rate
    \[
    \gamma_{k}^{f,i} = \frac{2\pi}{\hbar} p(\omega_k) \left| \left\langle 1_k, f \middle| \hbar \sum_{k} \lambda_k [\hat{b}_{k}^{+}, \hat{a}^{+}, \hat{b}_{k}] \right| 0, i \right\rangle \right|^2
    \]
  - In the dispersive limit the transmon states acquire only a small photonic component, it means we could treat it as a perturbation
  - After first order perturbation calculation, we get
    \[
    \gamma_{k}^{f,i} = \frac{2\pi}{\hbar} p(\omega_k) \lambda_k^2 \frac{g_{f,i}^2}{\left(\omega_{f,i} - \omega_r\right)^2}
    \]
  - We thus get modified $T_1 \sim 16 \mu$s
Dielectric losses

In insulating materials, especially amorphous SiO$_2$, it would affect the electric fields associated with the qubits and cause energy relaxation.

In particular, in the transmon design the shunting capacitance $C_B$ offers the possibility to accumulate a large percentage of the electric fields in a well controlled spatial region with favorable substrates,
- Quasiparticle tunneling
  - Following the arguments by Lutchyn et al.
    \[ N_{qp} = 1 + \frac{3\sqrt{2}}{2} N_e \frac{\sqrt{\Delta k_B T}}{E_F} e^{-\Delta/k_B T} \]
  - valid for temperatures small compared to the superconducting gap \( \Delta \)
  - \( T_1 \sim 1s \) in this channel when temperature is below 100mK
There is an intentional coupling between the SQUID loop and the flux bias (allowing for the $E_j$ tuning) through a mutual inductance $M$

In addition, the entire transmon circuit couples to the flux bias via a mutual inductance $M'$

We choose to assume that the applied flux could be decomposed into external flux and a small noise term, $\Phi = \Phi_e + \Phi_n, \Phi_n \ll \Phi_e$
Relaxation time—Amplitude damping channel

- Flux coupling
  - Then we Taylor expand the Josephson Hamiltonian
    \[ H_J \rightarrow H_J + \Phi_n A, \quad A = \left. \frac{\partial H_J}{\partial \Phi} \right|_{\Phi_e} \]
  - In this way, we could write the dissipation rate (just transition rate)
    \[ \Gamma_1 = \frac{1}{\hbar^2} |\langle 1 | A | 0 \rangle|^2 S_{\Phi_n} (\omega) = \frac{1}{\hbar^2} |\langle 1 | A | 0 \rangle|^2 M^2 S_{I_n} (\omega) \]
  - \( T_1: 20ms \sim 1s \)
  - However, new research found that there is capacitive coupling that contribute much more to the decoherence time \( \mu s \)
Relaxation time---Amplitude damping channel

- Other sources
  - Coupling to spurious resonator modes
  - Pinning and unpinning of vortices
  - Bulk piezoelectricity
Dephasing time---Phase damping channel

- Analytical discussion
- Charge noise & Flux noise
- Critical current noise
- Quasiparticle tunneling
- $E_C$ noise
The energy shift of the eigenstates would cause a change of transition frequency.
Then it would lead to a decay in the off diagonal elements.
In formal theory,
\[ H_q = \frac{1}{2} \sum_{u=x,y,z} h_u(\{\lambda_i\}) \sigma_u, \quad \lambda_i = \lambda_i^0 + \delta \lambda_i \]
Then do Taylor Expansion, calculate the noise power by autocorrelation. We focus on the dephasing aspect, low-frequency noise in the \(\sigma_z\) component.
\[ H_q = \frac{\hbar \omega_{01}}{2} \sigma_z + \frac{1}{2} \sum_j \frac{\partial h_z(\{\lambda_i\})}{\partial \lambda_j} \delta \lambda_j \sigma_z + O(\delta \lambda^2) \]
Then do Taylor Expansion, calculate the noise power by autocorrelation, we get the law of off diagonal elements

$$\rho_{01}(t) = e^{i\omega_{01}t} \exp\left( -\frac{1}{2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} S_v(\omega) \frac{\sin^2(\omega t/2)}{(\omega/2)^2} \right)$$

When $t \gg t_c$,

$$\rho_{01}(t) = e^{i\omega_{01}t} \exp\left( -\frac{1}{2} |t| S_v(\omega = 0) \right)$$

Dephasing time $T_2 \sim 2/S_v(\omega = 0)$, Lorentzian lineshape
Dephasing time---Phase damping channel

- Charge noise
  - Small fluctuations regime
    \[ T_2 \sim \frac{\hbar}{A} \left| \frac{\partial E}{\partial n_g} \right|^{-1} \sim 8s \]
  - Slow charge fluctuations with large amplitudes
    \[ H_q = \frac{1}{2} \left( \hbar \omega_{01} + \frac{\epsilon_1}{2} \cos[2\pi n_g + 2\pi \delta n_g(t)] \right) \sigma_z \]
    \[ T_2 \sim \frac{4\hbar}{e^2 \pi |\epsilon_1|} \sim 0.4ms, \quad \text{Transmon in worst case} \]
    \[ T_2 \sim \left| \frac{\pi^2 A^2}{\hbar^2} \frac{\partial^2 E}{\partial n_g^2} \right|^{-1} = \frac{\hbar^2}{\pi^2 A^2} \frac{E_j}{64E_c} \sim 1\mu s, \quad \text{Cooper Pair Box at sweet point} \]
Flux noise

- Noise in the externally applied flux translates into fluctuations of the effective Josephson coupling energy $E_J$.

\[
T_2 \sim \left| \frac{\pi^2 A^2}{\hbar^2} \frac{\partial^2 E}{\partial \Phi^2} \right|^{-1} \approx 3.6 \text{ms}, \quad \text{at integer flux quanta, sweet point}
\]
Critical current noise

Critical current noise is likely to be the limiting dephasing mechanism. Such rearrangements in the junction directly influence the critical current and hence the Josephson coupling energy

\[ E_j = I_c \hbar / 2e. \]

\[ T_2 \sim \frac{\hbar}{A} \left| \frac{\partial E}{\partial I_c} \right|^{-1} \sim 35 \mu s \]

An improvement of a factor of 2
Quasiparticle tunneling
- In CPB case, $\Gamma_{2}^{qp} = \Gamma^{qp} N_{qp}$, consider a jump from sweet point to half charge point
- But in transmon regime, due to large ratio of $E_J/E_C$, the charge dispersion is exponentially flat so that transition frequency variations due to a single charge are minimal.

$E_C$ noise
- A loophole?
- Fluctuations in the effective capacitances of the circuit.
- No current reports of this noise source.
# Dephasing time---Phase damping channel

<table>
<thead>
<tr>
<th>Noise source</th>
<th>1/f amplitude</th>
<th>Transmon $E_J/E_C=85$</th>
<th>CPB $E_J/E_C=1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge</td>
<td>$A=10^{-4} - 10^{-3}e$ [51]</td>
<td>400 000</td>
<td>1 000$^a$</td>
</tr>
<tr>
<td>Flux</td>
<td>$A=10^{-6} - 10^{-5}\Phi_0$ [52,54]</td>
<td>3 600 000$^a$</td>
<td>1 000 000$^a$</td>
</tr>
<tr>
<td>Critical current</td>
<td>$A=10^{-7} - 10^{-6}I_0$ [53]</td>
<td>35 000</td>
<td>17 000</td>
</tr>
</tbody>
</table>

$^a$These values are evaluated at a sweet spot (i.e., second-order noise).

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Koch et al. - 2007 - Charge-insensitive qubit design derived from the Cooper pair box
References


Thank You!