Implementation of the Deutsch-Josza Algorithm on an ion-trap quantum computer

S. Gulde et Al.

Przemyslaw Kotara – HU Berlin (Ger)

Tijl Schmelzer - U Ghent (Be)

Outline

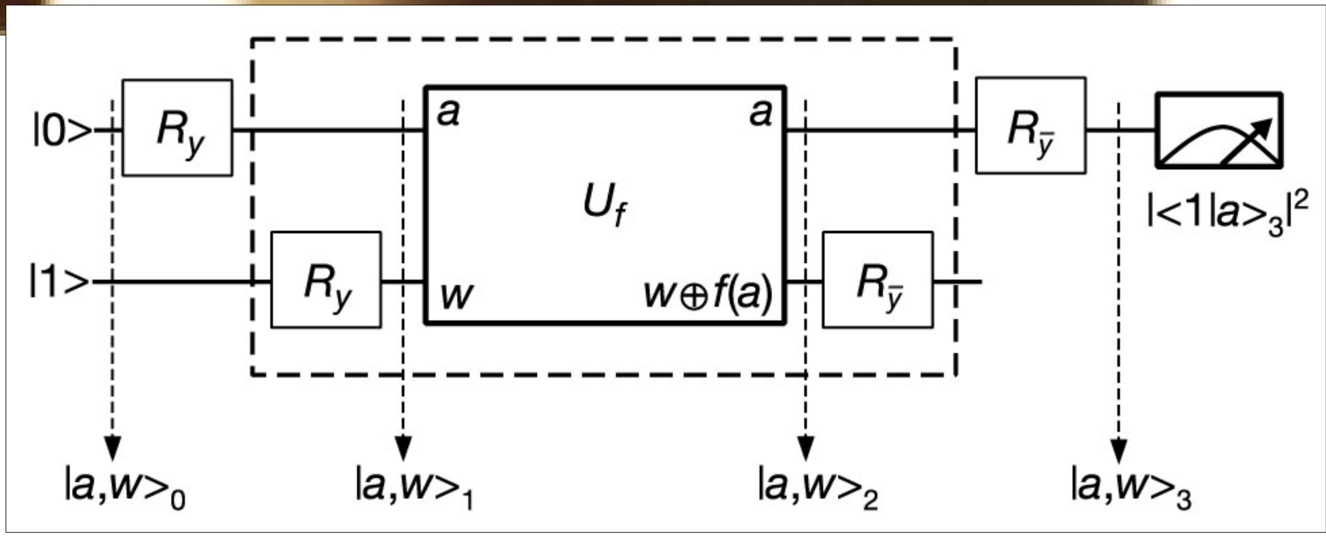
- Theory:
  - Problem/Motivation
  - The algorithm
    - Quantum Circuit
    - Deutsch algorithm
    - Deutsch-Jozsa algorithm
- Experiment:
  - Experimental setup
  - Error sources
  - Results
- References

Problem/Motivation

- Deutsch-Jozsa algorithm is a possibility for computing global properties of certain functions in exp. less time than any class. algorithm
- goal → determine the global property if a function is constant or balanced
- conventional deterministic algorithm takes $2^{n-1} + 1$ evaluations of f in the worst case
- Deutsch-Jozsa quantum algorithm produces an answer that is always correct with just 1 evaluation off
- Implementation serves to demonstrate the potential of ion traps for quantum computing

The Algorithm

Quantum Circuit



- Upper qubit (upper line) gives information which side of the coin
- Lower qubit (lower line) is an auxiliary working qubit
- R are rotations which create the superposition's
- U_f is an unitary operation
- Measurement of $|\langle 1|a\rangle_3|^2$ yields information if f is balanced or constant

The Algorithm

Deutsch Algorithm

$$1)\quad Input: \qquad |a,w\rangle_0 = |01\rangle = |0\rangle|1\rangle$$

$$2)\quad (H \otimes H): \qquad |a,w\rangle_1 = \frac{1}{2}\left(|0\rangle + |1\rangle\right)\left(|0\rangle - |1\rangle\right)$$

$$3)\quad U_f: \qquad |a,w\rangle_2 = \begin{cases} \pm\frac{1}{2}\left(|0\rangle + |1\rangle\right)\left(|0\rangle - |1\rangle\right) & for \quad f(0) = f(1) \\ \pm\frac{1}{2}\left(|0\rangle - |1\rangle\right)\left(|0\rangle - |1\rangle\right) & for \quad f(0) \neq f(1) \end{cases}$$

$$4)\quad (H \otimes 1): \qquad |a,w\rangle_3 = \begin{cases} \pm\frac{1}{\sqrt{2}}|0\rangle\left(|0\rangle - |1\rangle\right) & for \quad f(0) = f(1) \\ \pm\frac{1}{\sqrt{2}}|1\rangle\left(|0\rangle - |1\rangle\right) & for \quad f(0) \neq f(1) \end{cases}$$

$$5)\qquad = \pm\frac{1}{\sqrt{2}}|f(0) \oplus f(1)\rangle\left(|0\rangle - |1\rangle\right)$$

- The state of the first qubit shows if f is constant or balanced

- Explanation of step 3:

$$|x\rangle\left(|0\rangle-|1\rangle\right)=|x\rangle|0\rangle-|x\rangle|1\rangle\longrightarrow|x\rangle|0\oplus f(x)\rangle-|x\rangle|1\oplus f(x)\rangle$$

$$=|x\rangle|f(x)\rangle-|x\rangle|\neg f(x)\rangle$$

$$=\begin{cases}|x\rangle|0\rangle-|x\rangle|1\rangle & for \quad f(x)=0\\ |x\rangle|1\rangle-|x\rangle|0\rangle & for \quad f(x)=1\end{cases}$$

$$=(-1)^{f(x)}|x\rangle\left(|0\rangle-|1\rangle\right)$$

- If lx> itselve is a superpostiton, we have:

$$(-1)^{f(x)}|x\rangle=$$

$$(-1)^{f(0)}|0\rangle+(-1)^{f(1)}|1\rangle=\begin{cases} f(0)=f(1): & \begin{Bmatrix} f(0)=0: & +|0\rangle+|1\rangle \\ f(0)=1: & -|0\rangle-|1\rangle \end{Bmatrix} & =\pm\left(|0\rangle+|1\rangle\right) \\ f(0)\neq f(1): & \begin{Bmatrix} f(0)=0: & +|0\rangle-|1\rangle \\ f(0)=1: & -|0\rangle+|1\rangle \end{Bmatrix} & =\pm\left(|0\rangle-|1\rangle\right) \end{cases}$$

The Algorithm

Deutsch-Josza Algorithm

- It's easy to expand the algorithm to n qubit's:
- Initial state with n qubits is: $\left|a,w\right\rangle_0 = \left|\vec{0},1\right\rangle = \left|0\right\rangle_1 \left|0\right\rangle_2 \ldots \left|0\right\rangle_{n-1} \left|0\right\rangle_n \left|1\right\rangle_{n+1}$
- Algorithm is very similar to Deutsch Algorithm
- But applying the n-qubit Hadamard transformation to initial state:

$$H_{\vec{x}} = \prod_{i=1}^{n} H_i$$

- Final state of the n-qubit algorithm

$$\left|a,w\right\rangle_3 = \frac{1}{2^n} \sum_{\vec{z}} \sum_{\vec{x}} (-1)^{\vec{x}\cdot\vec{z}+f(x)} \left|\vec{z}\right\rangle \left(\frac{\left|0\right\rangle - \left|1\right\rangle}{\sqrt{2}} \right)$$

- Decide if f constant or balanced → measure population of ground state I0>

$$\frac{1}{2^n} \sum_{\vec{x}} (-1)^{f(\vec{x})} = \begin{cases} \pm 1 & for \quad f \quad const. \\ 0 & for \quad f \quad blanced \end{cases}$$

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Experimental Setup

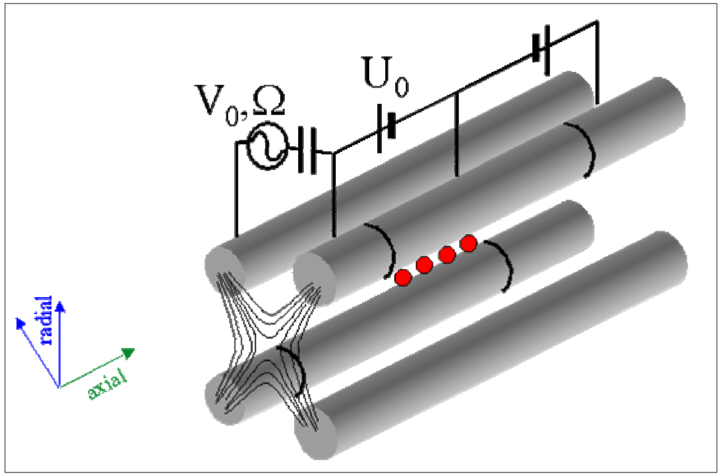
Overview

- ^{40}Ca+ ion in Linear Pauli-trap
- Lasercooling
- Ti-Sa-Laser for qubit-manipulations
  - Wavelength 729nm (linewidth<100Hz)
  - Acousto-optical modulator for freq-change and phaseshift
- Electron Shelving for electronic state Detection
  - 99,9% fidelity
  - 3ms detection time

Experimental Setup

Linear Pauli-trap

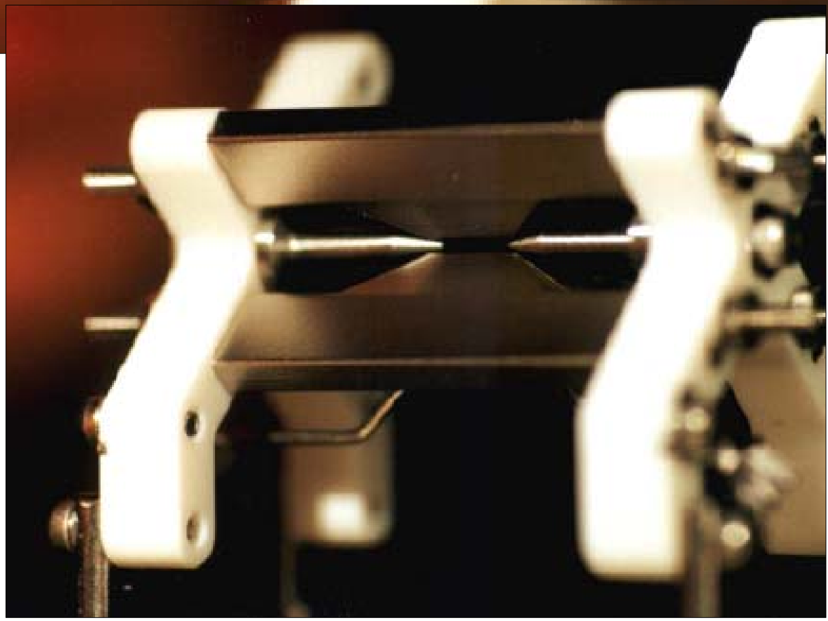
- Combination of static and alternating EM-fields → confine ions in an effective potential
- Field of ion trap = quadropole → vanishes at center & increases in all directions→ any deviations results in a net restoring force

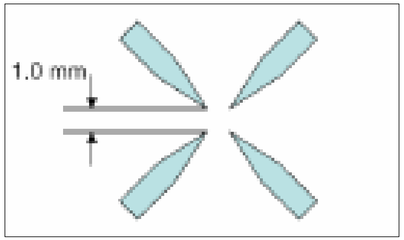


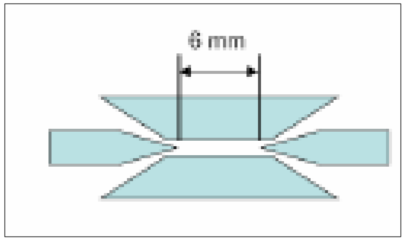
- Linear ion traps allows to assemble many ions in a linear chain, thus:
  - ➢ can be addressed by laser beams
  - ➢ equilibrium position is field free
- in contrast to classical non-linear Paul trap where trough coulomb repulsion ions are pushed away from field free point → micro motion

Experimental Setup

Linear Pauli-trap





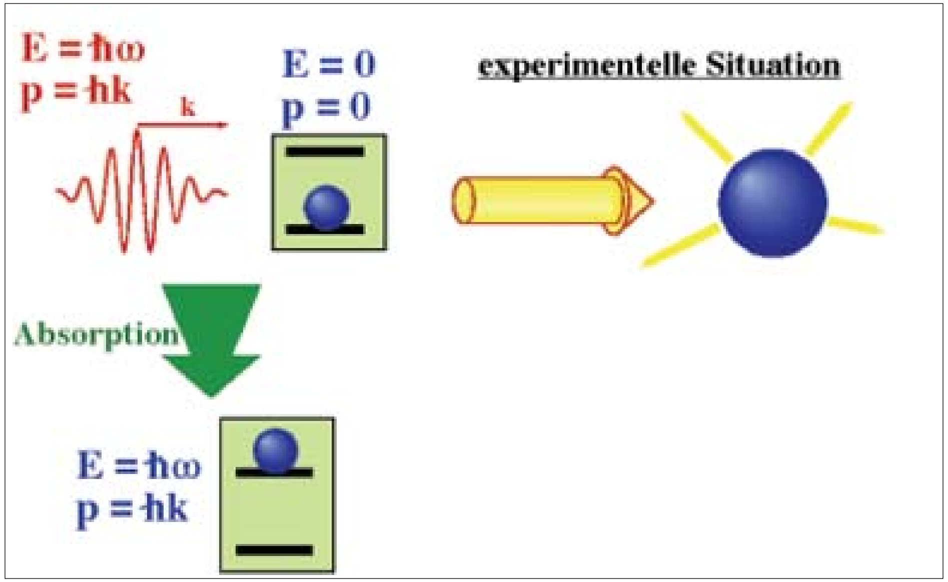


- $^{40}Ca+$ ions in Linear Pauli-trap
- $\omega_z = 2\pi * 1,7 MHz$

Experimental Setup

Doppler Laser Cooling

- Laser cooling relies on the transfer of momentum from photos → arrangement so that that forces push atoms in direction of the laser beam
- Momentum transfered ↔ photo is absorbed
- Emission in contrast of the absorption process is not directed → average effect of all emmission processes vanishes

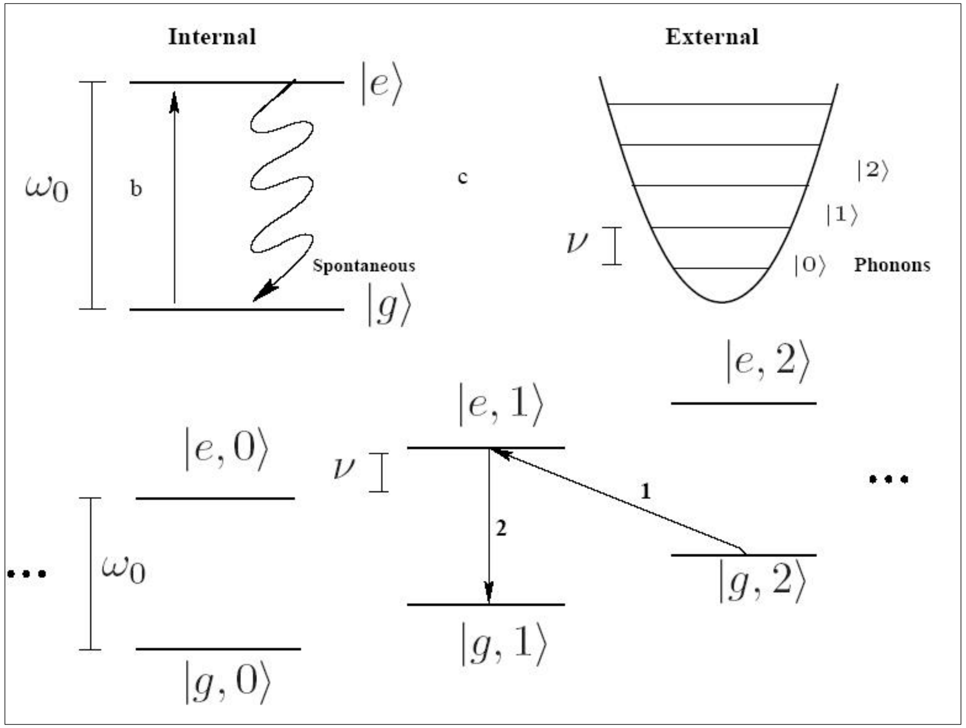


- One need high scattering rate because otherwise the change in velocity is too small
- Using lasers → scatter up to 10^8 photons per second →atom can be stopped over short distance

Experimental Setup

Sideband Laser Cooling

- Doppler cooling yield average vibrational quantum numbers $n_z \approx 20$
  → further cooling is achieved by sideband cooling
- Efficient laser cooling occurs when the frequency of the laser beam is tuned to the red sideband



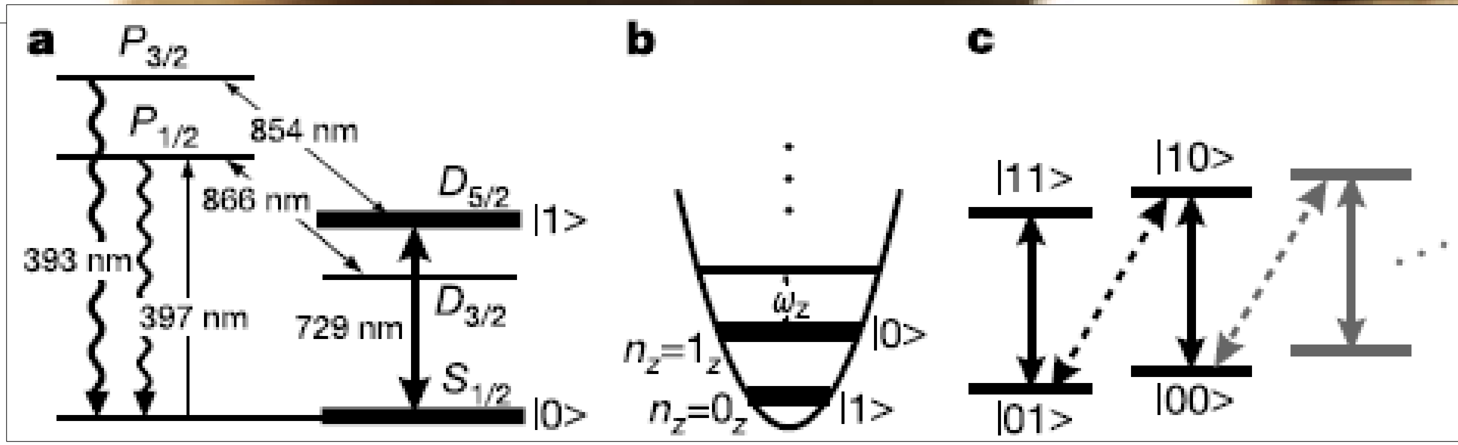
- In this case the atom undergoes the transition:

  |g,n> → |e,n-1>

- spontaneous emission occurs predominantly at the carrier frequency:
- |e,n-1> → |g,n-1>

Experimental Setup

QM energy levels



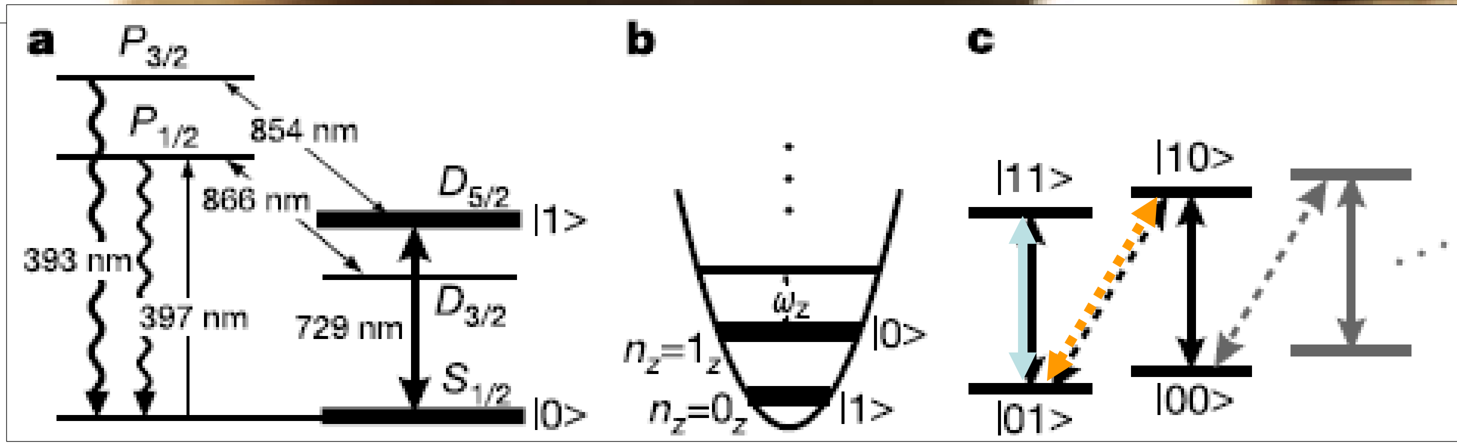
a. 1st Qubit a (Optical energylevels S½, D5/2)

b. 2nd Qubit w (Vibrational energylevels in ion trap 0_z, 1_z)

c. Combination law>

Experimental Setup

Qubit encoding



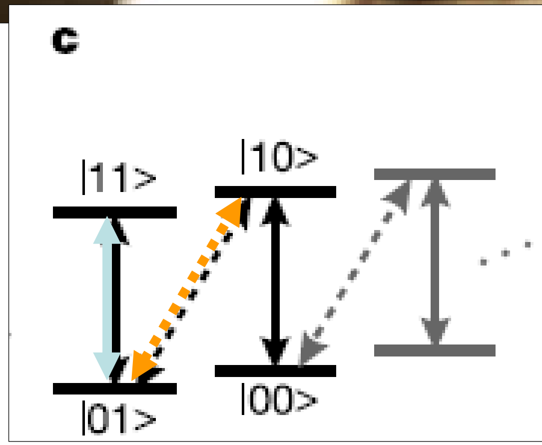
- Single-Qubit rotations R:
  - Carrier rotation
  - IS n_z> → ID n_z>
  - (729nm) Laser puls
- Double-Qubit rotations R+:
  - Transitions on the blue sideband
  - IS n_z> → ID n_z+1>
  - (729nm + ω_z) Laser puls

Experimental Setup

Qubit encoding

$$R(\theta,\phi) = \exp\left[i\frac{\theta}{2}\left(e^{i\phi}\sigma^{+} + e^{-i\phi}\sigma^{-}\right)\right]$$

$$R^{+}(\theta,\phi) = \exp\left[i\frac{\theta}{2}\left(e^{i\phi}\sigma^{+}b^{\dagger} + e^{-i\phi}\sigma^{-}b\right)\right]$$



- σ transitions between lS> and lD>
- *b* transitions between $l0_z$> and $l1_z$>
- θ ~ pulse duration
- φ phase between pulse and atomic polarization
- 2 important Rotations
  - $R_y = R(\pi/2, 0)$
  - $R_{\bar{y}} = R(\pi/2, \pi)$

Experimental Setup

Algorithm implementation

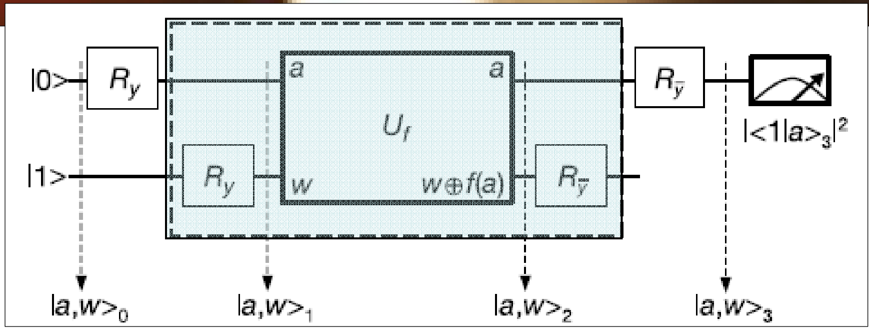


Table 1 **Truth table for the four possible functions**

	Constant functions		Balanced functions	
	Case 1	Case 2	Case 3	Case 4
$f(0)$	0	1	0	1
$f(1)$	0	1	1	0
$w \oplus f(a)$	ID	NOT	CNOT	Z-CNOT

Table 3 **Implementations of $R_{\bar{y}_w} U_{f_n} R_{y_w}$**

	Logic	Laser pulses
f_1	$R_{\bar{y}_w} R_{y_w}$	No pulses
f_2	$R_{\bar{y}_w}$ SWAP^{-1} NOT$_a$ SWAP R_{y_w}	$R^+\left(\frac{\pi}{\sqrt{2}},0\right)R^+\left(\frac{2\pi}{\sqrt{2}},\varphi_{\mathrm{SWAP}}\right)R^+\left(\frac{\pi}{\sqrt{2}},0\right)$ $R(\frac{\pi}{2},0)R(\pi,\frac{\pi}{2})R(\frac{\pi}{2},\pi)$ $R^+\left(\frac{\pi}{\sqrt{2}},\pi\right)R^+\left(\frac{2\pi}{\sqrt{2}},\pi+\varphi_{\mathrm{SWAP}}\right)R^+\left(\frac{\pi}{\sqrt{2}},\pi\right)$
f_3	$R_{\bar{y}_w}$ CNOT R_{y_w}	$R^+\left(\frac{\pi}{\sqrt{2}},0\right)R^+(\pi,\frac{\pi}{2})R^+\left(\frac{\pi}{\sqrt{2}},0\right)R^+(\pi,\frac{\pi}{2})$
f_4	$R_{\bar{y}_w}$ Z-CNOT R_{y_w}	$R(\pi,0)R^+\left(\frac{\pi}{\sqrt{2}},0\right)R^+(\pi,\frac{\pi}{2})R^+\left(\frac{\pi}{\sqrt{2}},0\right)R^+(\pi,\frac{\pi}{2})R(\pi,0)$

The rotation angle for $R^+(\theta,\varphi)$ is given for the $|10\rangle \rightarrow |01\rangle$ transition. θ and φ denote the pulse duration and phase, respectively. $\varphi_{\mathrm{SWAP}} = \arccos(\cot^2(\pi/\sqrt{2}))$.

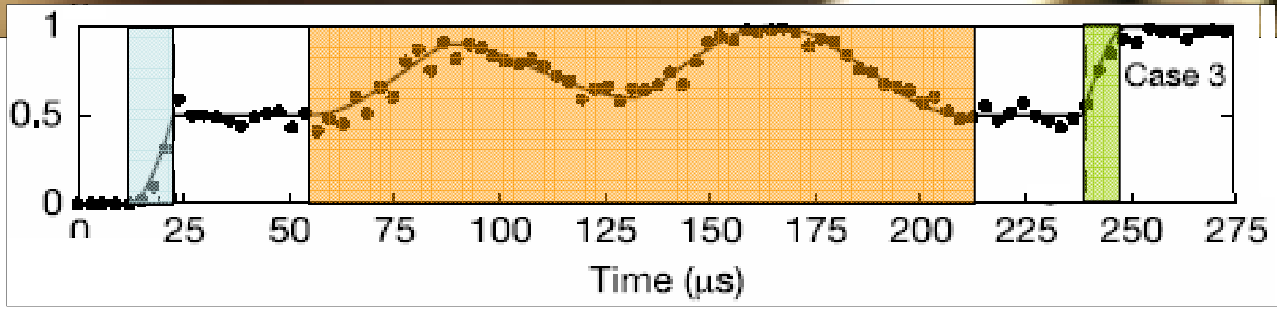
Experimental Setup

Startup

- Doppler lasercooling 2ms on S½ → P½
  - Result Vibrational quantumnumber n_z = 20
- Sideband Cooling 12ms
  - Result Vibrational Groundstate 0_z 99%
- Initialization by optically pump ion to S½
  - Result I01> = IS½ 0_z>

Experimental Setup

Case 3 as example



- 12 µs to 22 µs: R_{ya} carrier pulse
- 54 µs to 212 µs: $R_{\overline{yw}} U_{fn} R_{yw}$ blue sideband pulse on |aw>
  - The phase is switched at 87, 133 and 166 µs
- 240 to 250 µs: $R_{\overline{ya}}$ carrier pulse

Error sources

Phaseshift compensation

- 1 algorithm = several/many pulses
- Control relative phases precisely
- Unwanted shift has to be compensated

Error sources

Computational subspace

- Subspace {$|S\ 0_z\rangle$, $|D\ 0_z\rangle$, $|S\ 1_z\rangle$, $|D\ 1_z\rangle$}
- Transitions on the blue sideband
  - $|S\ n_z\rangle \rightarrow |D\ n_z+1\rangle$
  - $|S\ 1_z\rangle \rightarrow |D\ 2_z\rangle$ outside subspace
- Composite Pulses
  - Sequence of carrier and blue sideband pulses that constrain the system to the subspace

Results

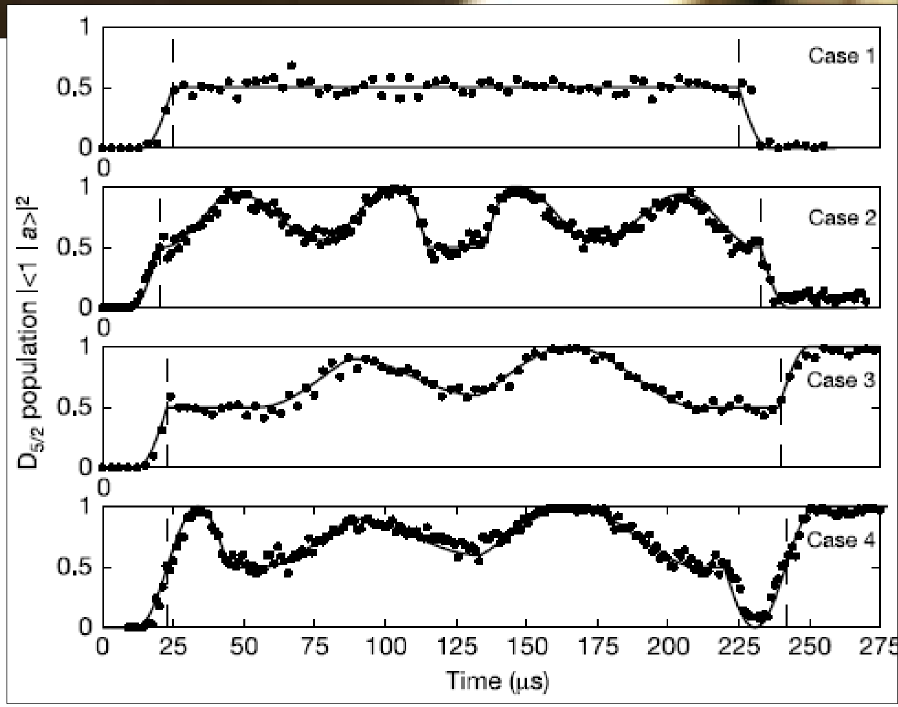
Measurements: Outcome

	Constant		Balanced	
	Case 1	Case 2	Case 3	Case 4
Expected $|\langle 1 | a\rangle|^2$	0	0	1	1
Measured $|\langle 1 | a\rangle|^2$	0.019(6)	0.087(6)	0.975(4)	0.975(2)
Expected $|\langle 1 | w\rangle|^2$	1	1	1	1
Measured $|\langle 1 | w\rangle|^2$	–	0.90(1)	0.931(9)	0.986(4)

- Fidelity $|\langle 1|a\rangle|^2$
  - Case 1,3,4 >97%
  - Case 2 >90%
- Error sources
  - Decoherence laser-atom phase
    - Mostly caused by ambient magnetic field fluctuations
  - Case 2 most complex pulse sequence
    - Higher laser power to speed up algorithm
    - → This reduces sensitivity to phase decoherence
    - → This causes off-resonant carrier excitation that limits fidelity

Results

Measurements: Evolution



Follow evolution of I<1Ia>I²

- Stop Pulse sequence anytime
- I<1Ia(t)>I² = Probability of finding ion in $D_{5/2}$ state
- Very small deviation of normal calculated ideal values (solid lines)

Results

Summary/Outlook

- High degree control over all relevant experimental parameters over long pulsesequences
  - Laser freq. and intensity, optical phases, and trap frequency ω_z
- Good procedure for the future
  - More complex algorithms
  - Scaling to multiple qubits
- Light shift compensation important for scaling
  - Ion heavier → higher laser intensities for sideband transitions which increases light shifts
- All gate operations possible
- Possible $^{43}Ca^+$ instead of $^{40}Ca^+$ with potentially longer coherence time

References

- Quantum Computing: A Short Courses form Theory to Experiment, ***J.Stolze and D.Suter***
- Verschränkte Systeme: Die Quantenphysik auf neuen Wegen, ***J. Andretsch***
- Seminar Quanteninformationsverarbeitung Vortrag Nr.4 Quantengatter & Algorithmen WS 02/03, ***S. Bauer, TU München***
- Quantum Computation and Quantum Information, ***M. Nielsen and I. Chuang***
- QSIT Lectures
- www.wikipedia.com