A SIX IONS SCHROEDINGER CAT
A presentation held by

Pierre Joris & Fabien Wildhaber

on the paper:

Creation of a six-atom 'Schrödinger' cat state,

(Boulder group)

Outline

• Implementation of a Six-ions Schrödinger Cat
  ➔ Hyperfine structure
  ➔ Trap
  ➔ Laser cooling
  ➔ Schrödinger cat

• Some Theory...
  ➔ Criteria for complete entanglement

• Measurements and Results
  ➔ Application of the criteria to measured data
Implementation of a Six-ions Schrödinger Cat
Implementation of a Six-ions Schrödinger Cat

- Qubits = Hyperfine levels of $^9\text{Be}^+$ corresponding to $|\uparrow\rangle$ and $|\downarrow\rangle$

- Paul trap: ions trapped in a RF-field

- Cooling and initialization of the 'cat state' using laser pulses
Hyperfine Structure: from QM

- Interaction between nuclear spin and dipolar moment of the electron
- \( F = \) total angular momentum
  \( m_F = \) projection on the \( z \)-axis
- Qubits states:
  \[ |\downarrow\rangle = |F = 2, m_F = -2\rangle \]
  \[ |\uparrow\rangle = |F = 1, m_F = -1\rangle \]
Paul Trap

- Oscillating RF Field
  => harmonic oscillations of the ions
  => Confinement
Laser Cooling

- 'Red Sideband' transition stimulated by laser: ... 2→1 → 0
- Decay by spontaneous emission: ↑ → ↓
- Each ion tends to the ground state of the harmonic oscillator |↓, 0⟩
- Notation for N ions: |↑↓...↑⟩ for the ground state: |↓↓...↓⟩≡|N, ↓⟩
Schrödinger Cat States

• Schrödinger cat state =
  Equal superposition of 2 maximally different quantum states

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} (\text{State 1} + \text{State 2}) \]
Schrödinger Cat States (2)

- Generated by rotation in the phase space of the ground state $|N, \downarrow \rangle = |\downarrow, \downarrow, ... , \downarrow \rangle$
- Implemented by action of two detuned laser beams stimulating a Raman transition (phase gate $U_N$)

$$\begin{align*}
|N \text{ Cat} \rangle &= U_N |N, \downarrow \rangle = \\
&= \frac{1}{\sqrt{2}} \left( |N, \uparrow \rangle + |N, \downarrow \rangle \right)
\end{align*}$$
Some
Theory...
Some Theory ...

- Proofs of complete entanglement
  - Fidelity
  - Witness operator
  - Condition over density matrix
Proofs of entangled state: Fidelity

- Fidelity $F$ is the simplest measure of how close the generated state $|\Psi_N\rangle$ is to $|N\text{ Cat}\rangle$

$$F_{N\text{ Cat}} \equiv |\langle \Psi_N | N\text{ Cat} \rangle|^2$$

if maximally entangled cat state $F_{N\text{ Cat}} = 1$

- Not sufficient to fully characterize entanglement
Proofs of entangled state: The Witness Operators

- Type of operators, for which \( \langle W \rangle < 0 \) => complete entanglement (i.e. each ion entangled with all others)

- In this paper we use

\[
W \equiv 1 - 2 |NCat\rangle \langle NCat| \tag{1}
\]

that has the important property:

\[
\langle W \rangle \leq 1 - 4 |C_{\downarrow N;\uparrow N}| \tag{2}
\]

where \( C_{\downarrow N;\uparrow N} = \langle \psi_N | \downarrow, N \rangle \langle \uparrow, N | \psi_N \rangle \) is one component of the density matrix (\(2^6 \times 2^6\))
Proofs of entangled state: The Witness Operators (2)

- Type of operators, for which $\langle W \rangle < 0 \Rightarrow$ complete entanglement (i.e. each ion entangled with all others)

- In this paper we use

$$W \equiv 1 - 2 \mid NCat \rangle \langle NCat \mid$$

that has the important property:

$$\langle W \rangle \leq 1 - 4 \mid C_{\downarrow N; \uparrow N} \mid$$ (2)

Consequence of (1) and (2):

$$\mid C_{\downarrow N; \uparrow N} \mid > 0.25 \Rightarrow \text{complete entanglement} \quad \text{(A)}$$
Proofs of entangled state: condition over density matrix

- Another condition for complete entanglement:

\[ 2|C_{\downarrow N;\uparrow N}| > \max (P_j + P_{\bar{j}}) \Rightarrow \text{compl. entanglement} \]  

where

\[ P_j = |\langle \Psi_N | j \rangle|^2 \]  
is the probability to measure \(|j\rangle\),

with \(|j\rangle\) a basis vector different from \(|N, \uparrow\rangle, |N, \downarrow\rangle\)

i.e.

\[ |j\rangle = |\downarrow \downarrow \downarrow \downarrow \uparrow \rangle, \ldots, |\downarrow \downarrow \uparrow \downarrow \uparrow \uparrow \rangle, \ldots, |\uparrow \uparrow \uparrow \uparrow \uparrow \downarrow \rangle \]

\[ |\bar{j}\rangle = |\uparrow \uparrow \uparrow \uparrow \uparrow \downarrow \rangle, \ldots, |\uparrow \downarrow \uparrow \uparrow \downarrow \downarrow \rangle, \ldots, |\downarrow \downarrow \downarrow \downarrow \uparrow \rangle \]
Measurements and Results
Measurements and Results

- Determination of population by fluorescence
- Calibration
- Determination of the density matrix components $C_{\downarrow N, \uparrow N}$
- Proof of the complete entanglement of the generated states
Determination of population by fluorescence and calibration

- Main observable to which we have access is the fluorescence of the down states (no fluorescence in the up state)
- Calibration carried out by statistical interpretation of fluorescence measurement of the known states $|N, \downarrow\rangle, N=0, \ldots, 6$
Determination of the $C_{\downarrow N; \uparrow N}$ 's

- We need the $C_{\downarrow N; \uparrow N}$ to verify entanglement conditions A and B
- By use of 'decoding' phase gate $U_{N, \Phi}$ (implemented by laser pulses) on the generated state $|\Psi_N\rangle$

we get a new **decoded** state:

$$|\Psi_\Phi\rangle = U_{N, \Phi} |N \text{ Cat}\rangle$$

$$= -i \sin\left(\frac{N}{2} \Phi\right) |\downarrow, N\rangle + i^{N+1+\xi} \cos\left(\frac{N}{2} \Phi\right) |\uparrow, N\rangle$$
Determination of the $C_{\downarrow N; \uparrow N}$'s (2)

- Fluorescence measurement over decoded states

=> Sinusoidal oscillations as function of $\phi$ (the parameter of $U_N, \phi$)

Normalized fluorescence vs. decoherence phase $\Phi$
Determination of the \( C_{\downarrow N;\uparrow N} \)'s (3)

- We have

\[
C_{\downarrow N;\uparrow N} \geq \frac{1}{2} \frac{A}{A_{\text{max}}}
\]

\( A_{\text{max}} \): Amplitude in perfect cat state case

\( A \): Effective amplitude

- Experimentally:

\[
|C_{\downarrow 4;\uparrow 4}| \geq 0.349 \quad |C_{\downarrow 5;\uparrow 5}| \geq 0.264 \quad |C_{\downarrow 6;\uparrow 6}| \geq 0.210
\]
Proof of entanglement (N=4,5)

- Recalling condition (A)

\[ |C_{\downarrow N, \uparrow N}| > 0.25 \implies \text{compl. entanglement} \]

- And since experimentally we got:

\[ |C_{\downarrow 4, \uparrow 4}| \geq 0.349 \]
\[ |C_{\downarrow 5, \uparrow 5}| \geq 0.264 \]
\[ |C_{\downarrow 6, \uparrow 6}| \geq 0.210 \]

we conclude that the states with N=4,5 are definitely completely entangled (for N=6 it is not sure at this point)
Proof of entanglement for N=6

• We rewrite now condition B:

\[ |C_{\downarrow N;\uparrow N}| > \max(P_j) \Rightarrow \text{compl. entanglement} \]

where the \( P_j \) are determined by fluorescence measurements over the state \( |\Psi_N\rangle \)

• Experimentally we have for N=6:

\[ |C_{\downarrow 6;\uparrow 6}| = 0.210 > \max(P_j) = 0.119 \]

• Conclusion: our \( |\Psi_6\rangle \) state is proved to be entangled too
References


• Boulder group homepage: http://tf.nist.gov/ion/index.htm


• QSIT Lecture Notes, Wallraff A.
QUESTIONS