
A SIX IONS
SCHROEDINGER
CAT

# *A SIX IONS SCHROEDINGER CAT*

*A presentation held by*

Pierre Joris & Fabien Wildhaber

*on the paper :*

<u>Creation of a six-atom 'Schrödinger' cat state,</u>

D. Leibfried, E. Knill, S. Seidelin, J. Britton, R. B. Blakestad, J. Chiaverini, D. B. Hurne, W. M. Itano, J. D. Jost, C. Langer, R. Ozeri, R. Reichle & D. J. Wineland
(**Boulder group**)

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# Outline

- Implementation of a Six-ions Schrödinger Cat
  - Hyperfine structure
  - Trap
  - Laser cooling
  - Schrödinger cat
- Some Theory...
  - Criteria for complete entanglement
- Measurements and Results
  - Application of the criteria to measured data

# Implementation of a Six-ions Schrödinger Cat

# Implementation of a Six-ions Schrödinger Cat

- Qubits = Hyperfine levels of $^{9}Be^{+}$ corresponding to $|\uparrow\rangle$ and $|\downarrow\rangle$

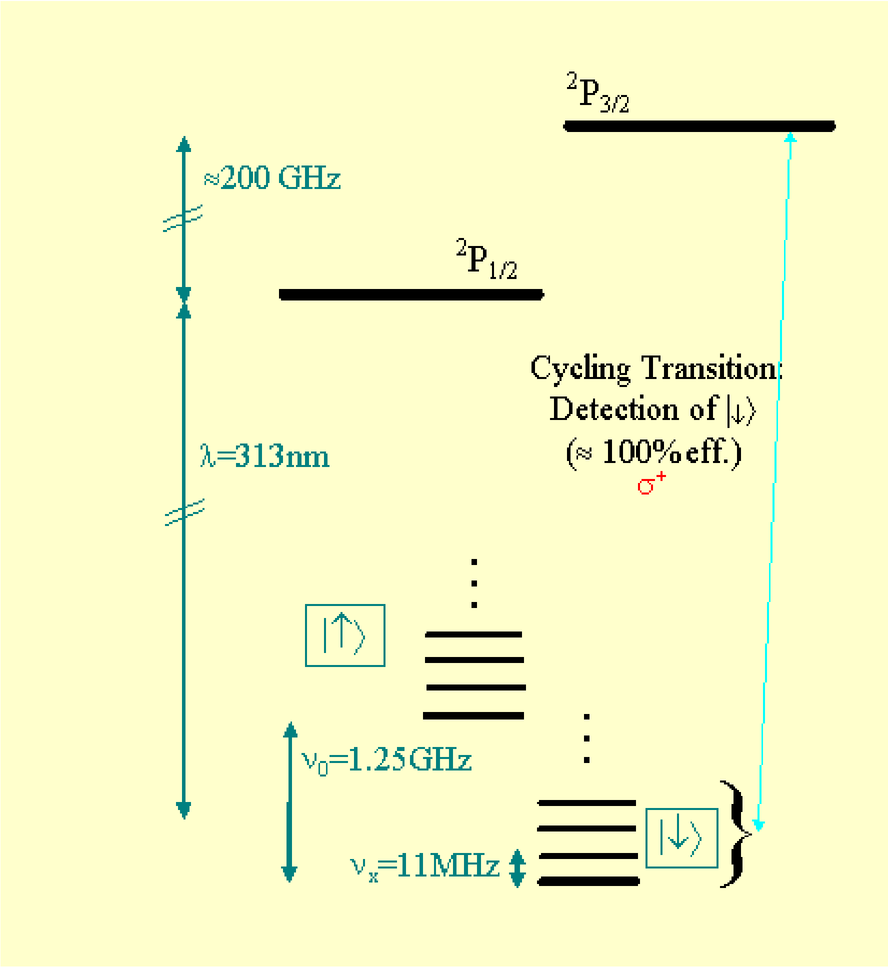
- Paul trap : ions trapped in a RF-field

- Cooling and initialization of the 'cat state' using laser pulses

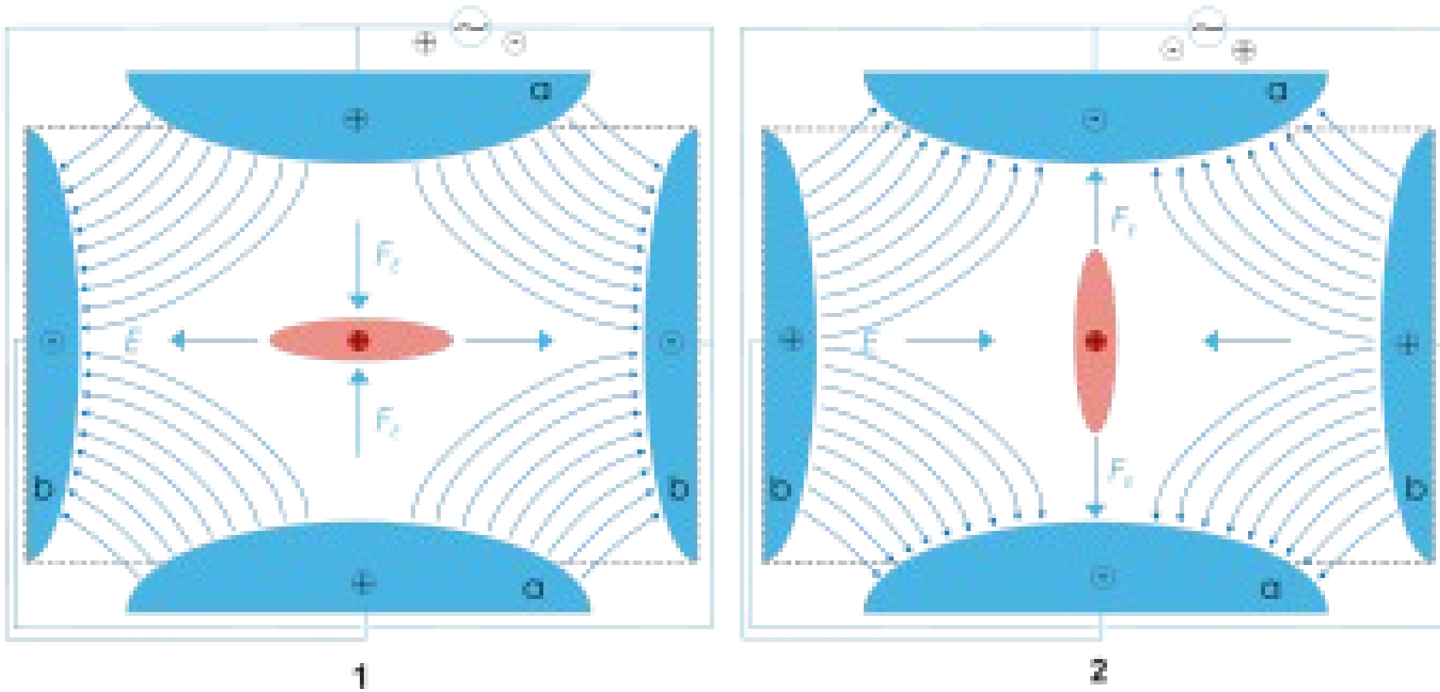
# Hyperfine Structure : from QM

- Interaction between nuclear spin and dipolar moment of the electron
- F = total angular momentum $m_F$ = projection on the z-axis
- Qubits states:

$$|\downarrow\rangle = |F=2, m_F=-2\rangle$$
$$|\uparrow\rangle = |F=1, m_F=-1\rangle$$



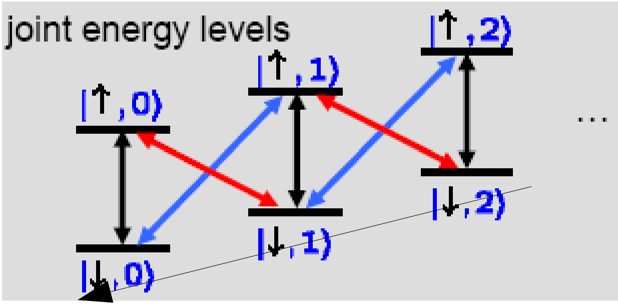
# Paul Trap



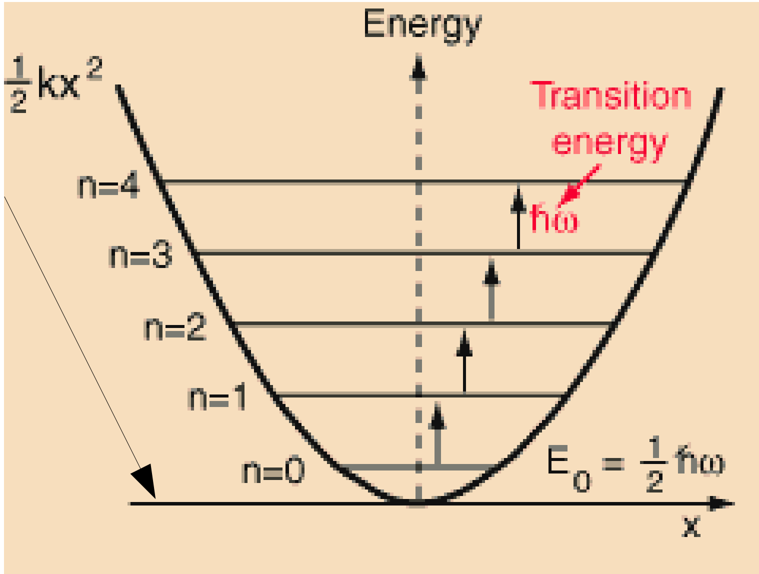

- Oscillating RF Field
  => harmonic oscillations of the ions
  => Confinement

# Laser Cooling

- 'Red Sideband' transition stimulated by laser: ... 2 → 1 → 0




- Decay by spontaneous emission: $\uparrow \rightarrow \downarrow$




- Each ion tends to the ground state of the harmonic oscillator $|\downarrow, 0\rangle$
- Notation for N ions: $|\uparrow\downarrow\ldots\uparrow\rangle$
  for the ground state: $|\downarrow\downarrow\ldots\downarrow\rangle \equiv |N, \downarrow\rangle$

# Schrödinger Cat States

- Schrödinger cat state =
  Equal superposition of 2 **maximally different** quantum states

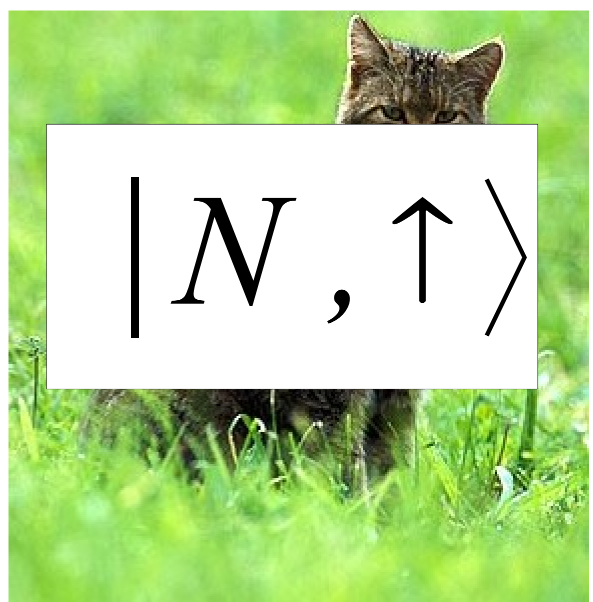
$|\Psi\rangle = \frac{1}{\sqrt{2}}($  $+$  $)$

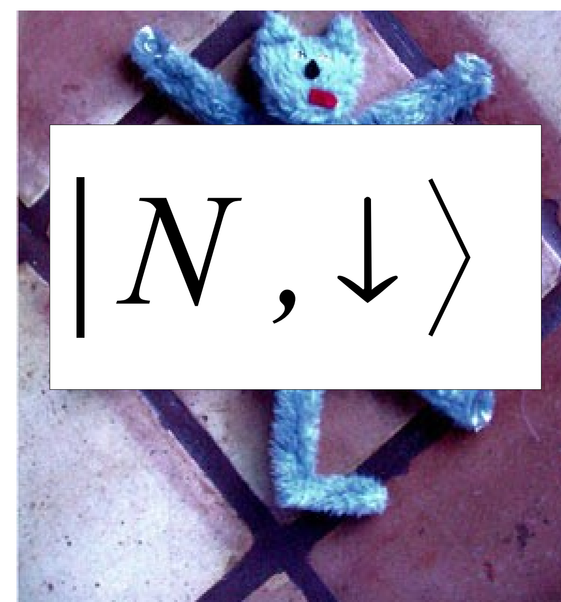
# Schrödinger Cat States (2)

- Generated by rotation in the phase space of the ground state $|N,\downarrow\rangle = |\downarrow,\downarrow,\ldots,\downarrow\rangle$
- Implemeted by action of two detuned laser beams stimulating a Raman transition (phase gate $U_N$)

$$|N\,Cat\rangle = U_N |N,\downarrow\rangle =$$

$$= \frac{1}{\sqrt{2}} \left( |N,\uparrow\rangle + |N,\downarrow\rangle \right)$$





# Some Theory…

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- Proofs of complete entanglement
  - Fidelity
  - Witness operator
  - Condition over density matrix

# Proofs of entangled state : Fidelity

- Fidelity F is the simplest measure of how close the generated state $|\Psi_N\rangle$ is to $|N\,Cat\rangle$

$$F_{N\,Cat} \equiv |\langle \Psi_N | N\,Cat \rangle|^2$$

  if maximally entangled cat state $F_{N\,Cat} = 1$

- Not sufficient to fully characterize entanglement

# Proofs of entangled state : The Witness Operators

- Type of operators, for which
  $\langle W \rangle < 0$ => complete entanglement (i.e. each ion entangled with all others)

- In this paper we use

$$W \equiv 1 - 2|NCat\rangle\langle NCat| \qquad (1)$$

that has the important property :

$$\langle W \rangle \leq 1 - 4|C_{\downarrow N;\uparrow N}| \qquad (2)$$

where $C_{\downarrow N;\uparrow N} = \langle \Psi_N | \downarrow, N \rangle \langle \uparrow, N | \Psi_N \rangle$
is one component of the density matrix ($2^6$x$2^6$)

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Consequence of (1) and (2) :

$$|C_{\downarrow N;\uparrow N}| > 0.25 \text{ =>complete entanglement} \qquad (A)$$

# Proofs of entangled state: condition over density matrix

- Another condition for complete entanglement:

$$2|C_{\downarrow N;\uparrow N}| > max(P_j + P_{\bar{j}})$$ => compl. entanglement (B)

where

$P_j = |\langle \Psi_N | j \rangle|^2$ is the probability to measure $|j\rangle$,

with $|j\rangle$ a basis vector different from $|N,\uparrow\rangle, |N,\downarrow\rangle$

i.e.

$$|j\rangle = |\downarrow\downarrow\downarrow\downarrow\downarrow\uparrow\rangle, \ldots, |\downarrow\downarrow\uparrow\downarrow\downarrow\uparrow\rangle, \ldots, |\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\rangle$$
$$|\bar{j}\rangle = |\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\rangle, \ldots, |\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow\rangle, \ldots, |\downarrow\downarrow\downarrow\downarrow\downarrow\uparrow\rangle$$

# Measurements and Results

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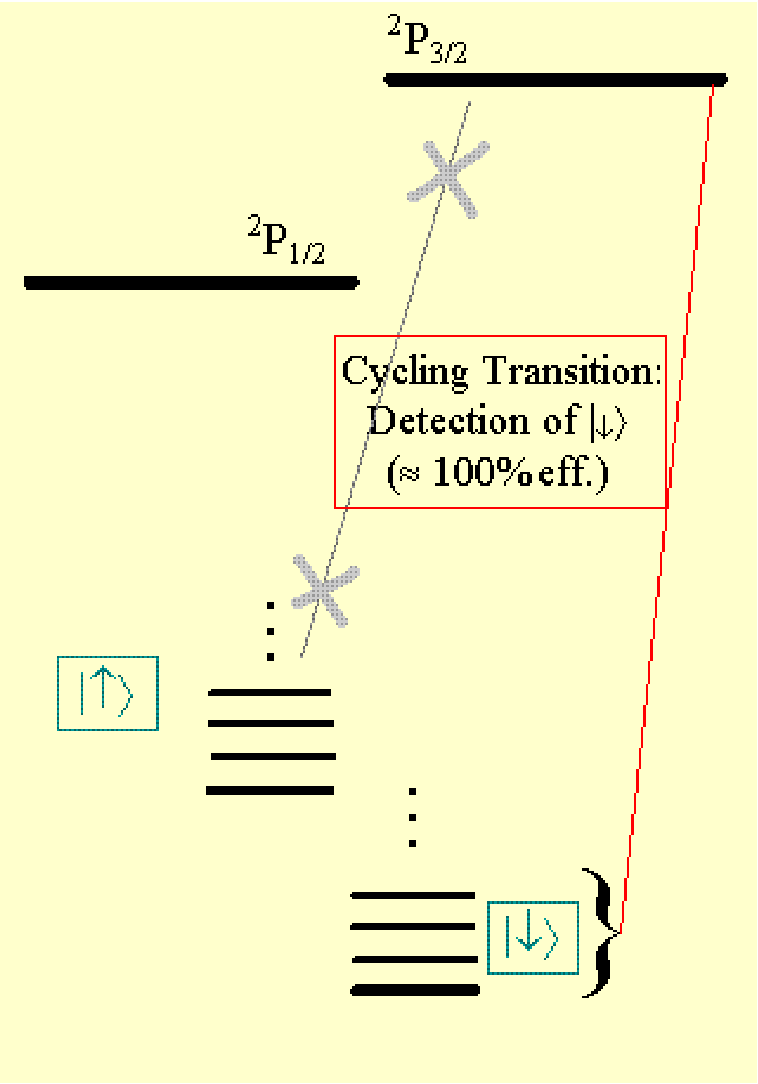
- Determination of population by fluorescence
- Calibration
- Determination of the density matrix components $C_{\downarrow N;\uparrow N}$
- Proof of the complete entanglement of the generated states

# Determination of population by fluorescence and calibration

- Main observable to which we have access is the fluorescence of the down states
(no fluorescence in the up state)




- Calibration carried out by statistical interpretation of fluorescence measurement of the known states

$$|N, \downarrow\rangle, N = 0, \ldots, 6$$

# Determination of the $C_{\downarrow N;\uparrow N}$ 's

- We need the $C_{\downarrow N;\uparrow N}$ to verify entanglement conditions A and B
- By use of 'decoding' phase gate $U_{N,\Phi}$ (implemented by laser pulses) on the generated state $|\Psi_N\rangle$

  we get a new **decoded** state:

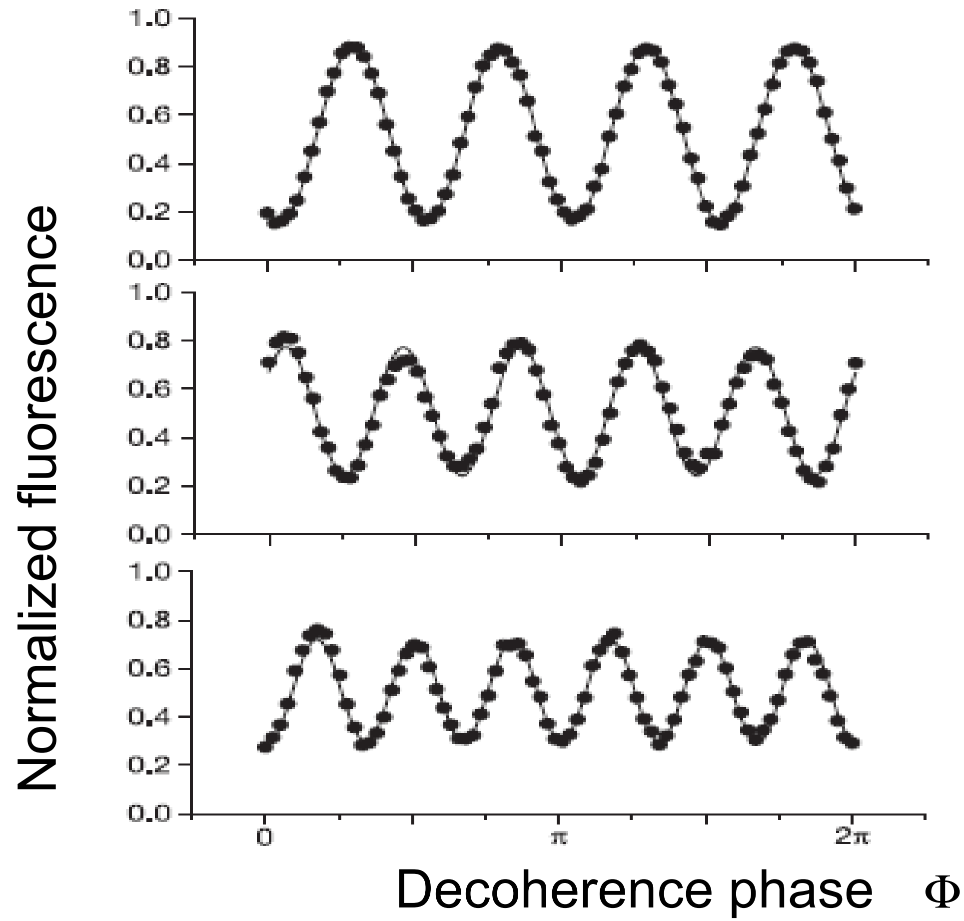
$$|\Psi_\Phi\rangle = U_{N,\Phi}|N\,Cat\rangle$$

$$= -i\sin(\frac{N}{2}\Phi)|\downarrow, N\rangle + i^{N+1+\xi}\cos(\frac{N}{2}\Phi)|\uparrow, N\rangle$$

# Determination of the $C_{\downarrow N;\uparrow N}$ 's (2)

- Fluorescence measurement over decoded states

  =>Sinusoidal oscillations as function of $\phi$ (the parameter of $U_{N,\Phi}$)



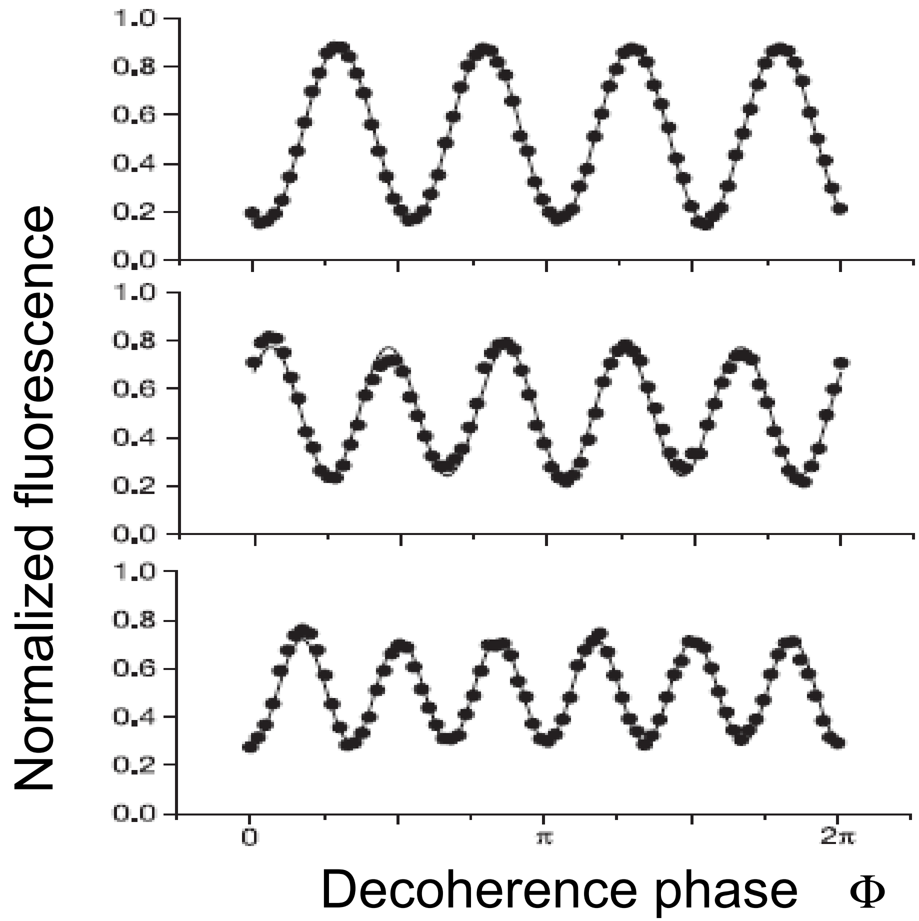
# Determination of the $C_{\downarrow N;\uparrow N}$'s (3)

- We have

$$C_{\downarrow N;\uparrow N} \geq \frac{1}{2} A / A_{max}$$

$A_{max}$: Amplitude in perfect cat state case

A : Effective amplitude




- Experimentally:

$$|C_{\downarrow 4;\uparrow 4}| \geq 0.349 \qquad |C_{\downarrow 5;\uparrow 5}| \geq 0.264 \qquad |C_{\downarrow 6;\uparrow 6}| \geq 0.210$$

# Proof of entanglement (N=4,5)

- Recalling condition (A)

$$\left|C_{\downarrow N;\uparrow N}\right| > 0.25 \quad \text{=>compl. entanglement}$$

- And since experimentally we got :

$$\left|C_{\downarrow 4;\uparrow 4}\right| \geq 0.349$$
$$\left|C_{\downarrow 5;\uparrow 5}\right| \geq 0.264$$
$$\left|C_{\downarrow 6;\uparrow 6}\right| \geq 0.210$$

we conclude that the states with N=4,5 are
**definitely completely entangled**
**(**for N=6 it is not sure at this point)

# Proof of entanglement for N=6

- We rewrite now condition B:

$$|C_{\downarrow N;\uparrow N}| > max(P_j)$$ => compl. entanglement

where the $P_j$ are determined by fluorescence measurements over the state $|\Psi_N\rangle$

- Experimentally we have for N=6:

$$|C_{\downarrow 6;\uparrow 6}| = 0.210 \;>\; max(P_j) = 0.119$$

- Conclusion : our $|\Psi_6\rangle$ state is **proved to be entangled** too

# References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QUESTIONS
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