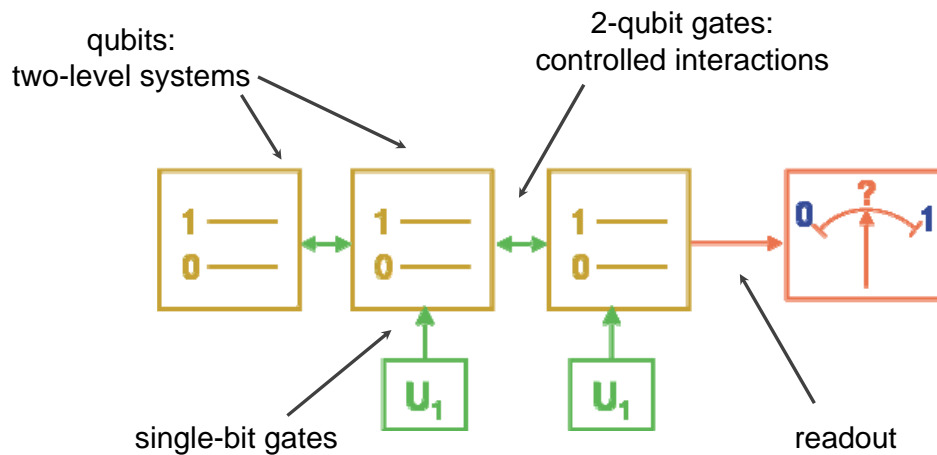


Generic Quantum Information Processor

The challenge:



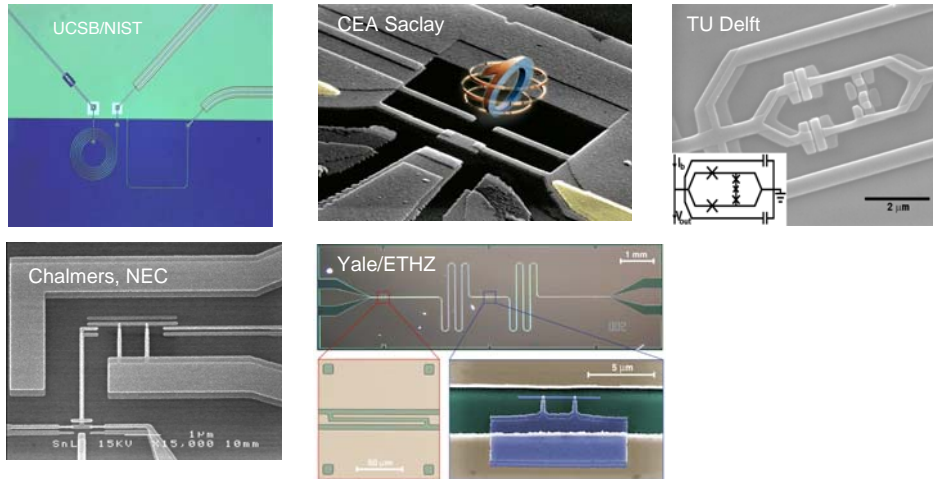
- Quantum information processing requires excellent qubits, gates, ...
- Conflicting requirements: good isolation from environment while maintaining good addressability

The 5 (+2) DiVincenzo Criteria for Implementation of a Quantum Computer:

in the standard (circuit approach) to quantum information processing (QIP)

- #1. A scalable physical system with well-characterized qubits.
- #2. The ability to initialize the state of the qubits to a simple fiducial state.
- #3. Long (relative) decoherence times, much longer than the gate-operation time.
- #4. A universal set of quantum gates.
- #5. A qubit-specific measurement capability.
- #6. The ability to interconvert stationary and mobile (or flying) qubits.
- #7. The ability to faithfully transmit flying qubits between specified locations.

Quantum Information Processing with Superconducting Circuits



Outline

- realization of superconducting qubits
- harmonic oscillators
- the current biased phase qubit
- the charge qubit
- qubit read-out
- single qubit control
- decoherence
- two-qubit gates

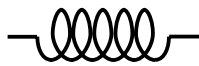
Some Basics ...

*... on how to construct qubits
using superconducting circuit elements.*

ETH

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Swiss Federal Institute of Technology Zurich

Building Quantum Electrical Circuits



inductor



capacitor



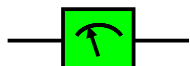
resistor



nonlinear element



voltage source



voltmeters

requirements for quantum circuits:

- low dissipation
- non-linear (non-dissipative elements)
- low (thermal) noise

a solution:

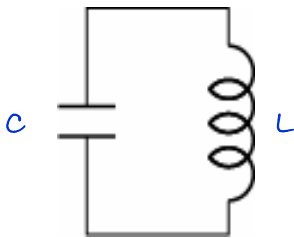
- use superconductors
- use Josephson tunnel junctions
- operate at low temperatures

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Superconducting Harmonic Oscillator

a simple electronic circuit:



- typical inductor: $L = 1 \text{ nH}$
- a wire in vacuum has inductance $\sim 1 \text{ nH/mm}$
- typical capacitor: $C = 1 \text{ pF}$
- a capacitor with plate size $10 \text{ }\mu\text{m} \times 10 \text{ }\mu\text{m}$ and dielectric AlOx ($\epsilon = 10$) of thickness 10 nm has a capacitance $C \sim 1 \text{ pF}$
- resonance frequency

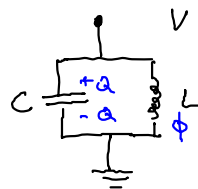
$$\frac{1}{2\pi\sqrt{LC}} \sim 5 \text{ GHz}$$

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Quantization of the electrical LC harmonic oscillator:

parallel LC oscillator circuit:



voltage across the oscillator:

$$V = \frac{Q}{C} = -L \frac{\partial I}{\partial t}$$

total energy (Hamiltonian):

$$H = \frac{1}{2} C V^2 + \frac{1}{2} L I^2 = \frac{1}{2} \frac{Q^2}{C} + \frac{1}{2} \frac{\Phi^2}{L}$$

with the charge Q stored on the capacitor

$$Q = VC$$

a flux Φ stored in the inductor

$$\Phi = LI$$

properties of Hamiltonian written in variables Q and Φ :

$$\frac{\partial H}{\partial Q} = \frac{Q}{C} = -L \frac{\partial I}{\partial t} = -\dot{\Phi}$$

$$\frac{\partial H}{\partial \Phi} = \frac{\Phi}{L} = I = \dot{Q}$$

Q and Φ are canonical variables

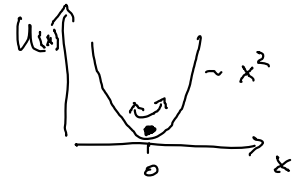
see e.g.: Goldstein, Classical Mechanics, Chapter 8, Hamilton Equations of Motion

Quantum version of Hamiltonian

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

with commutation relation

$$[\hat{\phi}, \hat{Q}] = i\hbar$$



compare with particle in a harmonic potential:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m \omega^2 \hat{x}^2$$

analogy with electrical oscillator:

- charge Q corresponds to momentum p

- flux ϕ corresponds to position x

$$[\hat{x}, \hat{p}] = [\hat{x}, i\hbar \frac{\partial}{\partial x}] = i\hbar$$

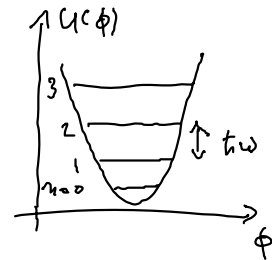
$$[\hat{\phi}, \hat{Q}] = [\hat{\phi}, -i\hbar \frac{\partial}{\partial \phi}] = i\hbar$$

Hamiltonian in terms of raising and lowering operators:

$$\hat{H} = \hbar \omega (a^\dagger a + \frac{1}{2})$$

with oscillator resonance frequency:

$$\omega = \frac{1}{\sqrt{LC}}$$



Raising and lowering operators:

$$a^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle ; \hat{a} |n\rangle = \sqrt{n} |n-1\rangle$$

$$a^\dagger a |n\rangle = n |n\rangle \quad \text{number operator}$$

in terms of Q and ϕ :

$$\hat{a} = \frac{1}{\sqrt{2\hbar Z_c}} (Z_c \hat{Q} + i \hat{\phi})$$

with Z_c being the characteristic impedance of the oscillator

$$Z_c = \sqrt{\frac{L}{C}}$$

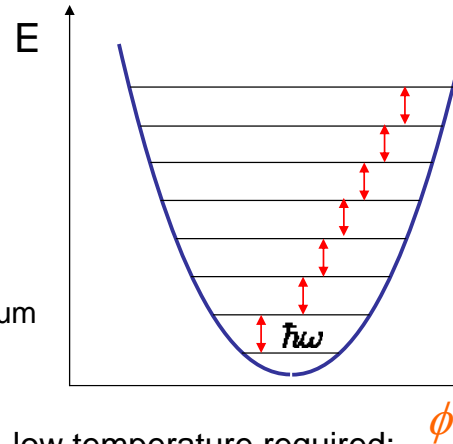
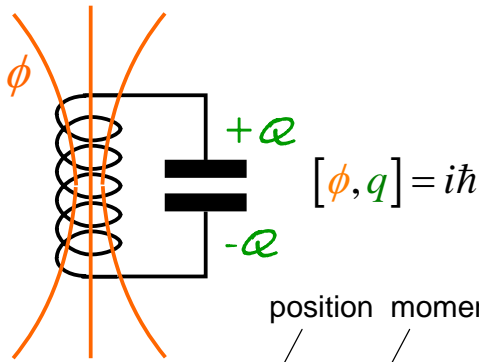
charge Q and flux ϕ operators can be expressed in terms of raising and lowering operators:

$$\hat{Q} = \sqrt{\frac{\hbar}{2Z_c}} (a + a^\dagger)$$

$$\hat{\phi} = \sqrt{\frac{2Z_c \hbar}{i}} (a - a^\dagger)$$

Exercise: Making use of the commutation relations for the charge and flux operators, show that the harmonic oscillator Hamiltonian in terms of the raising and lowering operators is identical to the one in terms of charge and flux operators.

Quantum LC Oscillator



Hamiltonian

$$H = \frac{\phi^2}{2L} + \frac{q^2}{2C}$$

$$\omega = 1/\sqrt{LC}$$

$$H = \hbar\omega \left(a^\dagger a + \frac{1}{2} \right)$$

low temperature required:

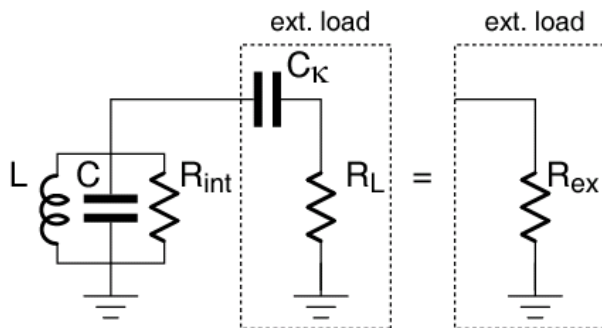
$$\hbar\omega \gg k_B T$$

10 GHz ~ 500 mK 20 mK

$$\langle n_{th} \rangle = \frac{1}{\exp(\hbar\omega/k_B T) - 1} \sim 10^{-11}$$

problem 1: **equally spaced energy levels (linearity)**

Dissipation in an LC Oscillator



internal losses: R_{int}
conductor, dielectric

external losses: R_{ext}
radiation, coupling

total losses

$$\frac{1}{R} = \frac{1}{R_{int}} + \frac{1}{R_{ext}}$$

impedance

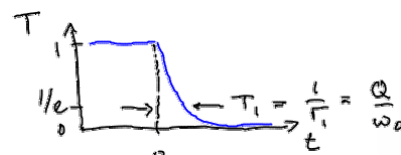
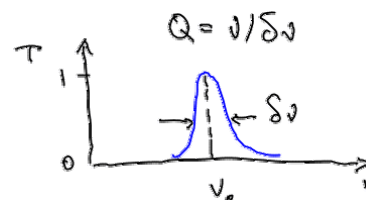
$$Z = \sqrt{\frac{L}{C}}$$

quality factor

$$Q = \frac{R}{Z} = \omega_0 RC$$

excited state decay rate

$$\Gamma_1 = \frac{\omega_0}{Q} = \frac{1}{RC}$$



problem 2: **internal and external dissipation**