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Oscillations about different axes:

$$H = \frac{\Omega}{2}\sigma_z + \varepsilon\cos(\omega t + \varphi)\sigma_x = \vec{m}(t)\cdot\vec{\sigma} \qquad \vec{m}(t) = \begin{pmatrix} \varepsilon\cos\omega t + \varphi \\ 0 \\ \frac{\Omega}{2} \end{pmatrix}$$

$$\vec{m}(t) = \frac{1}{2}\begin{pmatrix} \varepsilon\cos(\omega t+\varphi) \\ \varepsilon\sin(\omega t+\varphi) \\ \frac{\Omega}{2} \end{pmatrix} + \frac{1}{2}\begin{pmatrix} \varepsilon\cos(\omega t+\varphi) \\ -\varepsilon\sin(\omega t+\varphi) \\ \frac{\Omega}{2} \end{pmatrix}$$

RWA

$$H = \frac{\Omega}{2}\sigma_z + \frac{\varepsilon}{2}\Big[\underbrace{\cos(\omega t+\varphi)\sigma_x + \sin(\omega t+\varphi)\sigma_y}_{\circledast}\Big] + \frac{\varepsilon}{2}\Big[\cancel{\cos(\omega t+\varphi)\sigma_x - \sin(\omega t+\varphi)\sigma_y}\Big]$$

$$\boxed{UHU^\dagger - iU\dot{U}^\dagger|\phi\rangle = i\hbar\frac{d}{dt}|\phi\rangle}$$

Rotating Frame @ ωt: $U = e^{i\frac{\omega}{2}t\sigma_z}$ $\qquad U\dot{U}^\dagger = -i\frac{\omega}{2}\sigma_z$

$$U\sigma_x U^\dagger = \cos\omega t\,\sigma_x - \sin\omega t\,\sigma_y$$
$$U\sigma_y U^\dagger = \sin\omega t\,\sigma_x + \cos\omega t\,\sigma_y$$

$$\circledast = \cos(\omega t+\varphi)(\cos\omega t\,\sigma_x - \sin\omega t\,\sigma_y) + \sin(\omega t+\varphi)(\sin\omega t\,\sigma_x + \cos\omega t\,\sigma_y)$$

$$\begin{cases} \cos(\omega t+\varphi) = \cos\omega t\cos\varphi - \sin\omega t\sin\varphi \\ \sin(\omega t+\varphi) = \sin\omega t\cos\varphi + \cos\omega t\sin\varphi \end{cases}$$

$$= \sigma_x\Big(\cos\omega t\cdot(\cos\omega t\cos\varphi - \sin\omega t\sin\varphi) + \sin\omega t(\sin\omega t\cos\varphi + \cos\omega t\sin\varphi)\Big) =$$
$$+\ \sigma_y\Big(-\sin\omega t\,(\ \text{—}\ -\ \text{—}\ ) + \cos\omega t\,(\ \text{—}\ +\ \text{—}\ )\Big) =$$

$$= \sigma_x(\cos^2\omega t + \sin^2\omega t)\cos\varphi + \sigma_y\sin\varphi$$

$$\Rightarrow \tilde{H} = \frac{1}{2}(\Omega-\omega)\sigma_z + \frac{\varepsilon\cos\varphi}{2}\sigma_x + \frac{\varepsilon\sin\varphi}{2}\sigma_y =$$

$$= \frac{1}{2}(\Omega-\omega)\sigma_z + \frac{\Omega_x}{2}\sigma_x + \frac{\Omega_y}{2}\sigma_y = \frac{1}{2}\vec{m}\cdot\vec{\sigma} = \frac{1}{2}\alpha\,\vec{n}\cdot\vec{\sigma}$$

$$\alpha = |\vec{m}| = \sqrt{(\Omega-\omega)^2 + \Omega_x^2 + \Omega_y^2} \qquad \vec{n}\ \text{.. unit vector}$$

$$\vec{n} = \begin{pmatrix} \cos\varphi\sin\theta \\ \sin\varphi\sin\theta \\ \cos\theta \end{pmatrix}$$

$$\varphi = \arctan\frac{\Omega_y}{\Omega_x}$$

$\theta = \arccos\frac{\Omega-\omega}{\ldots}$

Ramsey Fringes:

*) create superposition state by a $\frac{\pi}{2}$ - pulse

*) free evolution for time t

*) 2^{nd} $\frac{\pi}{2}$ - pulse about different axis

State after the first $\frac{\pi}{2}$ pulse:

$$|g\rangle \rightarrow \left(\frac{\pi}{2}\right)_\phi : \frac{1}{\sqrt{2}}\left(|g\rangle - i e^{i\phi}|e\rangle\right)$$

$$U_{\phi_1} = e^{-i\frac{\alpha t}{2}\vec{n}_{\varphi_1}\vec{\sigma}} \quad \text{with} \quad \alpha = \frac{\pi}{2} \quad \vec{n} = \begin{pmatrix} \cos\varphi_1 \\ \sin\varphi_1 \\ \sim 0 \end{pmatrix}$$

$$U_{\left(\frac{\pi}{2}\right)\phi_1} = \mathbb{1}\cos\frac{\alpha t}{2} - i\vec{n}\cdot\vec{\sigma}\cdot\sin\frac{\alpha t}{2} = \frac{1}{\sqrt{2}}\left(\mathbb{1} - i\left(\cos\varphi_1\sigma_x + \sin\varphi_1\sigma_y\right)\right)$$

$$U_{\left(\frac{\pi}{2}\right)\phi_1}|g\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle - i\cos\varphi_1|e\rangle + \sin\varphi_1|e\rangle\right) =$$

$$= \frac{1}{\sqrt{2}}\left(|g\rangle - i e^{i\varphi_1}|e\rangle\right) = \Psi_1$$

Free precession: $\varepsilon = 0$: $\quad \alpha = |\Omega - \omega| \equiv \Delta \quad \vec{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$U_{free} = e^{-i\frac{|\Omega-\omega|t'}{2}\sigma_z}$$

$$U_{free}\, U_{\left(\frac{\pi}{2}\right)\phi_1}|g\rangle = e^{-i\frac{\Delta t'}{2}\sigma_z}\frac{1}{\sqrt{2}}\left(|g\rangle - i e^{i\varphi_1}|e\rangle\right) =$$

$$= \frac{1}{\sqrt{2}}\, e^{-i\frac{\Delta t'}{2}}|g\rangle - i e^{i\left(\frac{\Delta t'}{2} + \varphi_1\right)}|e\rangle \,\alpha$$

Second $\frac{\pi}{2}$ pulse about φ_2 axis

$$U_{\left(\frac{\pi}{2}\right)\phi_2}\Psi_2 = \frac{1}{2}\left(\mathbb{1} - i\left(\cos\varphi_2\sigma_x + \sin\varphi_2\sigma_y\right)\right)\left(e^{-i\frac{\Delta t'}{2}}|g\rangle - i e^{i\left(\frac{\Delta t'}{2} + \varphi_1\right)}\right.$$

$\sigma_x|g\rangle \rightarrow |e\rangle \qquad \sigma_y|g\rangle \rightarrow i|e\rangle$

$\sigma_x|e\rangle \rightarrow |g\rangle \qquad \sigma_y|e\rangle \rightarrow -i|g\rangle$

$$= \frac{1}{2}\Big[e^{-i\frac{\Delta t'}{2}}|g\rangle - i e^{i\left(\frac{\Delta t'}{2}+\varphi_1\right)}|e\rangle - i e^{-i\frac{\Delta t'}{2}}(\cos\varphi_2 + i\sin\varphi_2)|e\rangle - e^{i\left(\frac{\Delta t'}{2}+\varphi_1\right)}(\cos\varphi_2 - i\sin\varphi_2)|g\rangle \Big] =$$

$$= \frac{1}{2}\Big[|g\rangle \left(e^{-i\frac{\Delta t'}{2}} - e^{i\left(\frac{\Delta t'}{2}+\varphi_1-\varphi_2\right)} \right) - i|e\rangle \left(e^{i\left(\frac{\Delta t'}{2}+\varphi_1\right)} + e^{-i\left(\frac{\Delta t'}{2}-\varphi_2\right)} \right) \Big]$$

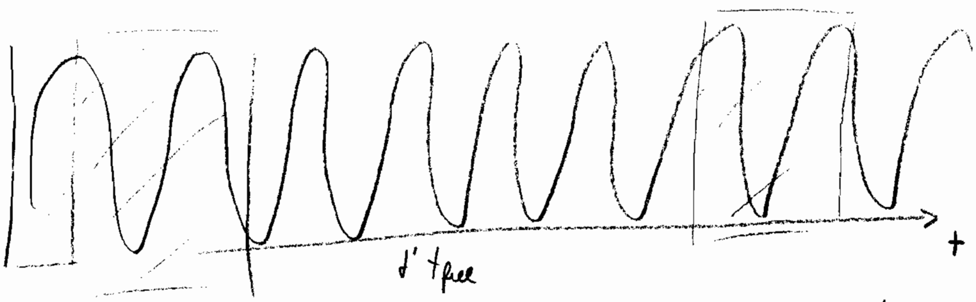
$$= \frac{1}{2}\Big[e^{i(\varphi_1-\varphi_2)/2}|g\rangle \left(\overbrace{e^{-i(\Delta t'+\varphi_1-\varphi_2)/2} - e^{i(\Delta t'+\varphi_1-\varphi_2)/2}}^{-2i\sin(\Delta t+\Delta\varphi)} \right) -$$

$$i e^{i(\varphi_1+\varphi_2)/2}|e\rangle \left(\underbrace{e^{i(\Delta t'+\varphi_1-\varphi_2)/2} + e^{-i(\Delta t'+\varphi_1-\varphi_2)/2}}_{2\cos(\Delta t+\Delta\varphi)} \right)$$

$$\propto \sin(\Delta t+\Delta\varphi)|g\rangle + e^{i\varphi_2}\cos(\Delta t+\Delta\varphi)|e\rangle$$

$\Rightarrow$ population changes with Δt and $\Delta\varphi$

one source with fixed phase



φ_1 $\quad$ $\varphi_1 + t_{\frac{E}{2}}\Delta$ $\quad$ $\varphi_2 = \varphi_1 + \underbrace{t_{\frac{E}{2}}\Delta}_{\ll 1} + t'\Delta$

$\Rightarrow$ change either t' or Δ for a change in φ_2

Spin Echo: basic idea: $U^{-1} U |\psi\rangle = |\psi\rangle$

$$U = e^{-\frac{i}{\hbar} H t} \Rightarrow U^{-1} = e^{\frac{i}{\hbar} H t} \Rightarrow \text{time reversal needed}$$

for free-precession Hamiltonian:

$H = \Omega \sigma_z$: change either Ω (possible for spin-$\frac{1}{2}$ in magnetic field)

or swap basis states: $|g\rangle \to |e\rangle$
$|e\rangle \to |g\rangle$

$$|\psi\rangle \to |\psi'\rangle = \sigma_x |\psi\rangle$$

$$\Rightarrow \hat{H} = U^+ H U = \Omega \sigma_x^+ \sigma_z \sigma_x = \Omega \sigma_x \sigma_z \sigma_x = -\Omega \sigma_z$$

σ_x .. π-pulse

Sequence: $U_{\frac{\pi}{2}} U_{free} U_{\pi} U_{free} U_{\frac{\pi}{2}} \cdot |\psi\rangle = |\psi\rangle$