Bloch equations:

$$H = -\mu\, \vec{\sigma} \cdot \vec{m}(t) \qquad \vec{B}(t) = \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix}$$

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] \qquad \rho = |\psi\rangle\langle\psi| = \frac{1}{2}(1 + \vec{r}\cdot\vec{\sigma})$$

$$\dot{\rho} = +\frac{i\mu}{\hbar}\left[\vec{\sigma}\cdot\vec{m}\cdot\frac{1}{2}(1+\vec{r}\vec{\sigma}) - \frac{1}{2}(1+\vec{r}\cdot\vec{\sigma})\cdot\vec{\sigma}\vec{m}\right] =$$

$$= +\frac{i\mu}{2\hbar}\left[(\vec{m}\cdot\vec{\sigma})(\vec{r}\cdot\vec{\sigma}) - (\vec{r}\cdot\vec{\sigma})(\vec{m}\cdot\vec{\sigma})\right]$$

using $(\vec{a}\cdot\vec{\sigma})(\vec{b}\cdot\vec{\sigma}) = (\vec{a}\cdot\vec{b})\,1 + i(\vec{a}\times\vec{b})\cdot\vec{\sigma}$

$$= +\frac{i\mu}{\hbar}\left(-i\,\vec{r}\times\vec{m}\right)\vec{\sigma}$$

$$= \frac{\mu}{\hbar}\left(\vec{r}\times\vec{m}\right)\cdot\vec{\sigma}$$

in terms of Bloch vector components:

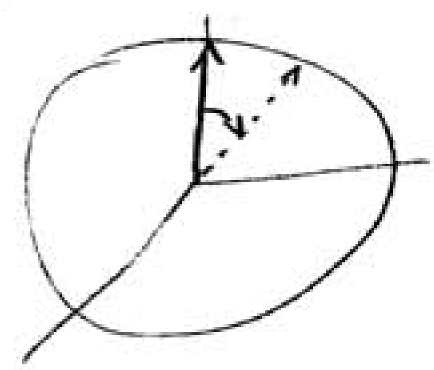
$$\dot{\vec{r}} = \mathrm{Tr}(\dot{\rho}\vec{\sigma}) = \overset{\gamma}{\left(\frac{\mu}{\hbar}\right)}(\vec{r}\times\vec{m}) \quad \dots \text{"Bloch equations"}$$

$m_y = 0$

$$\dot{r}_x = \gamma\, r_y m_z \qquad \text{e.g.: } \vec{r}(0) = \begin{pmatrix}0\\0\\1\end{pmatrix} \Rightarrow \dot{r}_y \neq 0$$

$$\dot{r}_y = \gamma\,(r_z m_x - r_x m_z)$$

$$\dot{r}_z = -\gamma\, r_y m_x$$



Problem: no relaxation to thermal equilibrium

e.g. $\vec{m} = m_z$, system in excited state: $(|r_z| = 1) \Rightarrow \dot{r} = 0$

introduce longitudinal and transversal relaxation

$$\dot{r}_x = \gamma\,(\vec{r}\times m)_x - \frac{r_x}{T_2}$$

$$\dot{r}_y = \gamma\,(\vec{r}\times m)_y - \frac{r_y}{T_2}$$

$$\dot{r}_z = \gamma\,(\vec{r}\times m)_z - \frac{r_z - r_z^s}{T_1}$$

r_z^s ... steady state (-1 for ground state)

now: with $r_z^s = -1$: $\quad \dot{r}_z = -(r_z+1)\cdot\frac{1}{T_1}$

$$\rightarrow r_z(t) = e^{-\frac{t}{T_1}}\left(r_z + r_z(0)\right) - 1$$

$\rightarrow T_1$: longitudinal relaxation time

e.g.: $r_x(0) = 1 \qquad \tilde{m} = m_z$; $T_1 = 0$

$$\left.\begin{aligned} \dot{r}_y &= -\gamma\, r_x m_z - \frac{r_y}{T_2} \\ \dot{r}_x &= \gamma\, r_y m_z - \frac{r_x}{T_2} \end{aligned}\right\} r_x(t), r_y(t) \propto e^{-\frac{t}{T_2}}$$

T_2: transverse relaxation time, dephasing