Superconducting Circuits II

Coupling Superconducting Qubits via a Cavity

Superconducting Circuits II Outline

- Cavity (circuit) QED

- Paper by DiCarlo et al.
  - Experimental setup
  - Realization of one-qubit and two-qubit gates
  - Creation of entangled states
  - Simple quantum algorithms
  - Readout

- Summary

Superconducting Circuits II

- Quantum computation requires efficient interaction between spatially separated qubits

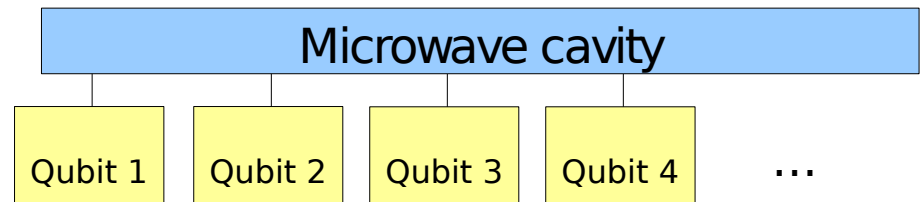   → need for a mobile qubit

Superconducting Circuits II

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    → need for a mobile qubit

- Natural choice = photons
    - Superconducting qubits – relatively large dipole moment
        → strong interactions with electromagnetic field

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- Cavity bus architecture

Microwave cavity				
Qubit 1	Qubit 2	Qubit 3	Qubit 4	…

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Microwave cavity				
Qubit 1	Qubit 2	Qubit 3	Qubit 4	...

- Theory to describe qubit/cavity interactions = cQED
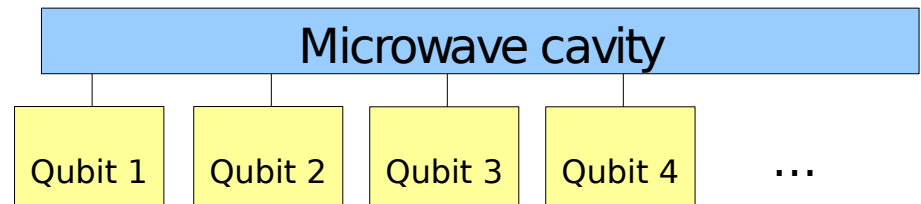
Cavity (circuit) QED

- described by the Jaynes-Cummings hamiltonian:

$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{1}{2}\hbar\omega_q \hat{\sigma}_z + \hbar g(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger \hat{\sigma}_-)$$

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- more qubits – Tavis-Cummings hamiltonian:

$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \sum_j \left(\frac{1}{2}\hbar\omega_q^{(j)} \hat{\sigma}_z^{(j)} + \hbar g^{(j)}(\hat{a}\hat{\sigma}_+^{(j)} + \hat{a}^\dagger \hat{\sigma}_-^{(j)}) \right)$$

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can be generalized to multi-level qubits (turns out to be useful)

ETH
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Swiss Federal Institute of Technology Zurich

Cavity (circuit) QED

- interaction terms in the Tavis-Cummings hamiltonian for two qubits

$$\hbar g^{(1)}(\hat{a}\hat{\sigma}_+^{(1)} + \hat{a}^\dagger\hat{\sigma}_-^{(1)}) + \hbar g^{(2)}(\hat{a}\hat{\sigma}_+^{(2)} + \hat{a}^\dagger\hat{\sigma}_-^{(2)})$$

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couples $|10\rangle_q \otimes |0\rangle_c \rightarrow |00\rangle_q \otimes |1\rangle_c$

couples $|00\rangle_q \otimes |1\rangle_c \rightarrow |01\rangle_q \otimes |0\rangle_c$

effective coupling $|10\rangle_q \leftrightarrow |01\rangle_q$

mediated by virtual photon exchange (does not need to be in resonance with the cavity)

Cavity (circuit) QED

- How can one observe coupling experimentally?

- Simple example: two level system

$$\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

E_2 ————————

E_1 ————————

Cavity (circuit) QED

- How can one observe coupling experimentally?
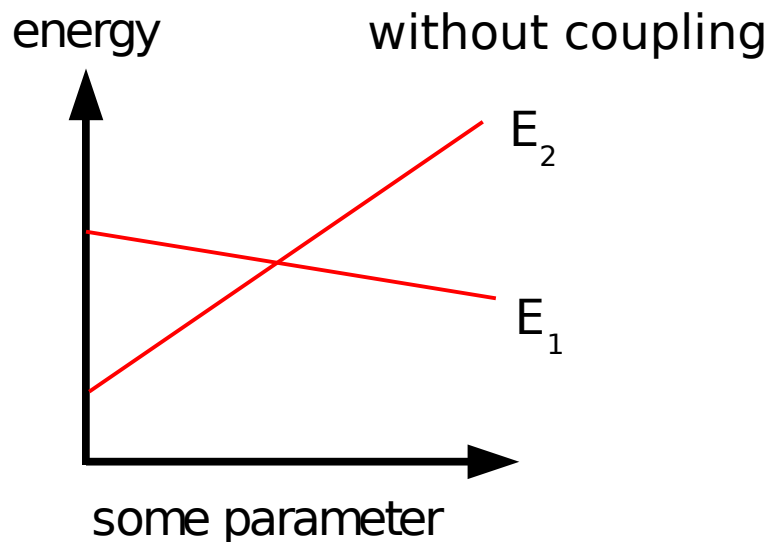
- Simple example: two level system

$$\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \longrightarrow \begin{pmatrix} E_1 & A \\ A & E_2 \end{pmatrix}$$

Cavity (circuit) QED

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- Simple example: two level system

$$
\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \longrightarrow \begin{pmatrix} E_1 & A \\ A & E_2 \end{pmatrix}
$$

$E_2 + \Delta$

E_2

E_1

Δ is always non-zero!

$E_1 - \Delta$

Cavity (circuit) QED

- How can one observe coupling experimentally?

- Simple example: two level system
  - significant change in the energy spectrum when coupling is turned on

energy   without coupling

E_2

E_1

some parameter
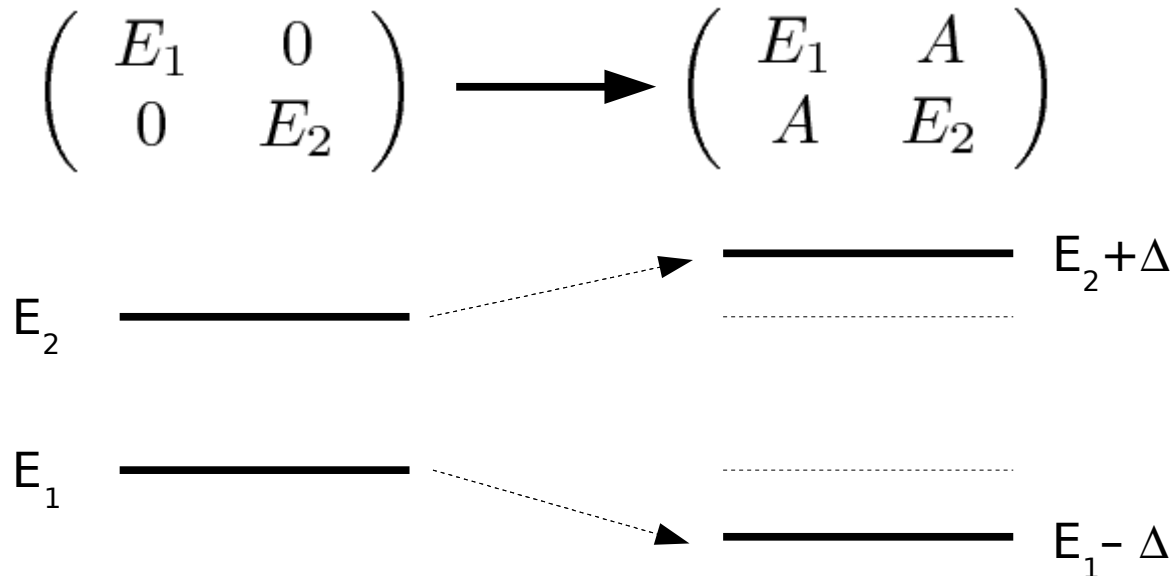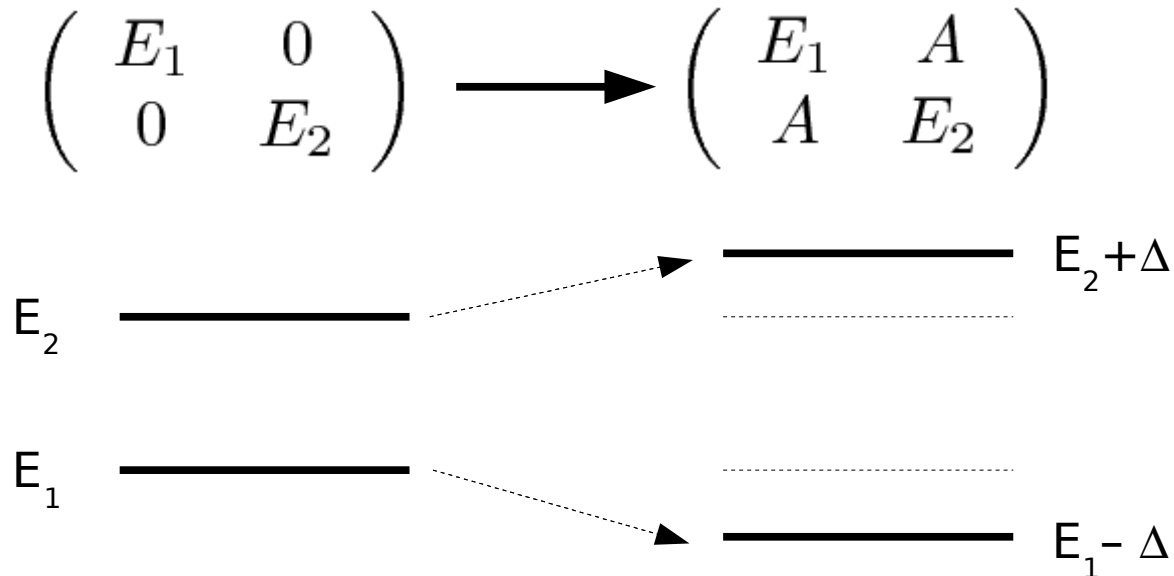
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avoided crossing

some parameter
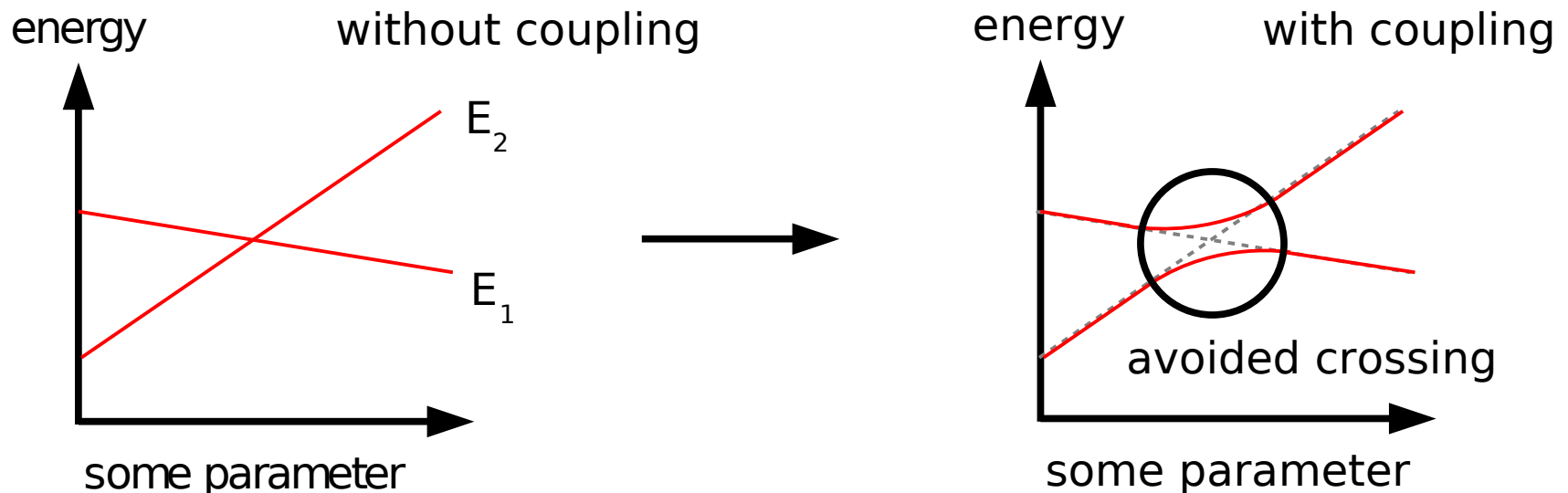
Cavity (circuit) QED

- avoided crossing in qubits coupled to a cavity observed by

  J. Majer *et al.*

  Nature **449**,

  443-447 (2007)
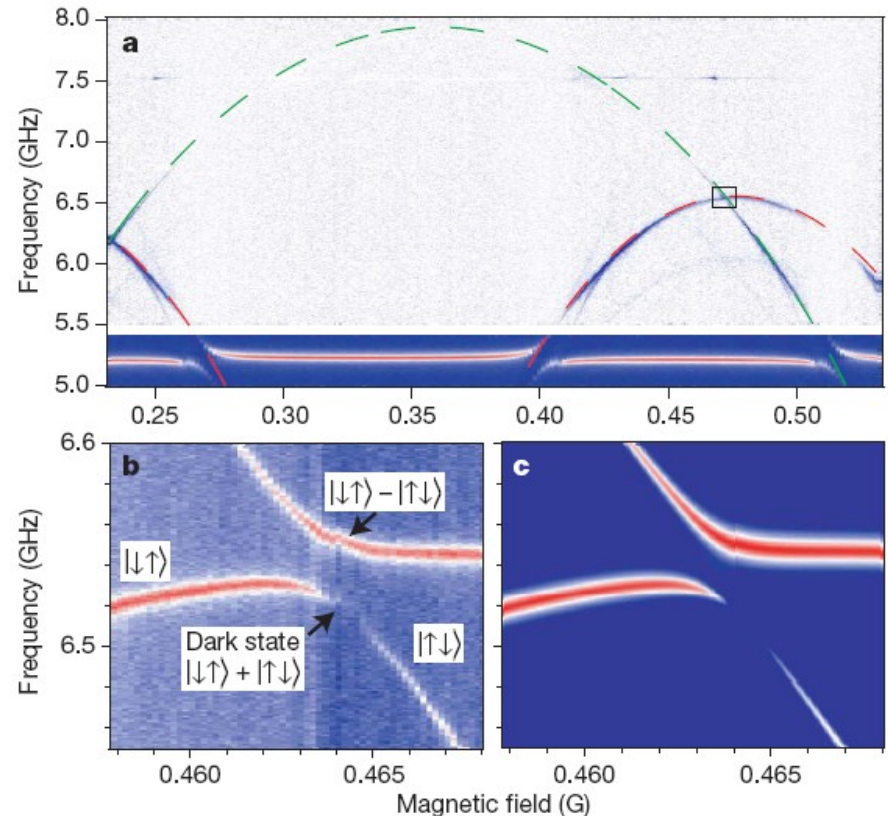
Demonstration of two-qubit algorithms with a superconducting processor

L. DiCarlo *et. al.* Nature **460**, 08121 (2009)

- Aim:
  - Couple two superconducting transmon qubits via a transmission line cavity
  - Create a two-qubit (C-phase) gate
  - Create entangled two-qubit states
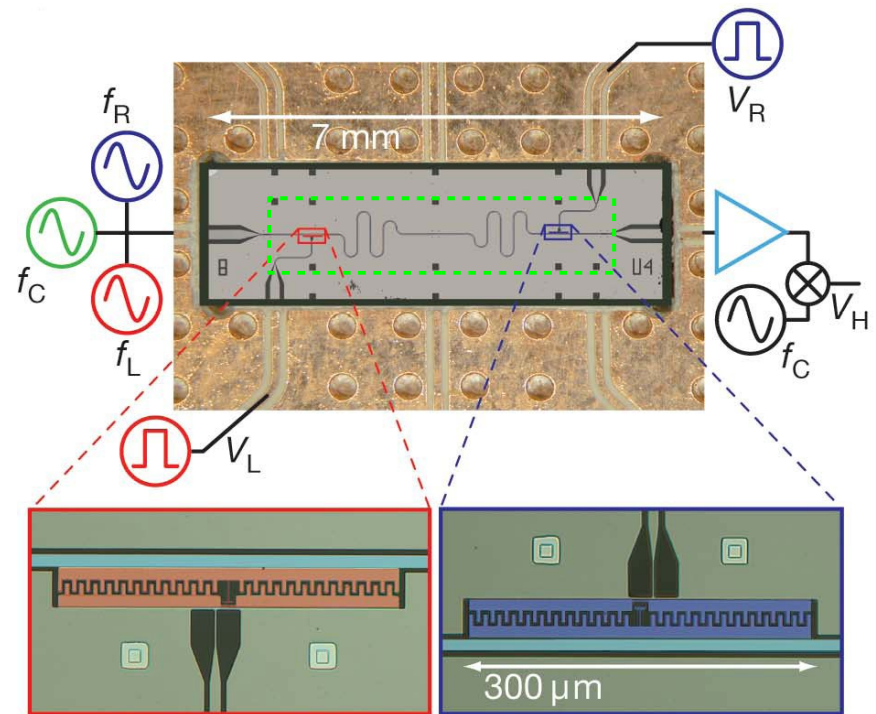 - Demonstrate simple two-qubit algorithms (Grover, Deutsch-Jozsa)

Experimental Setup

- superconducting circuit
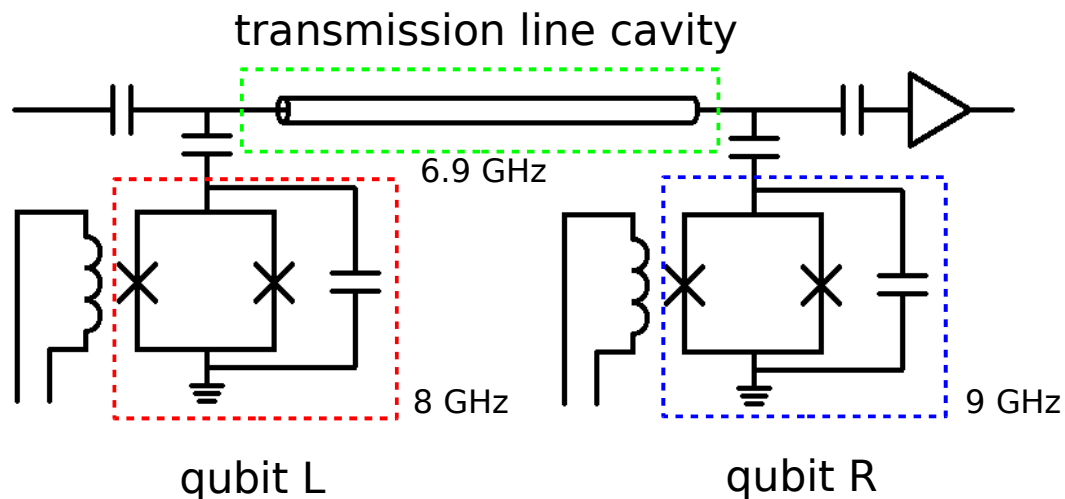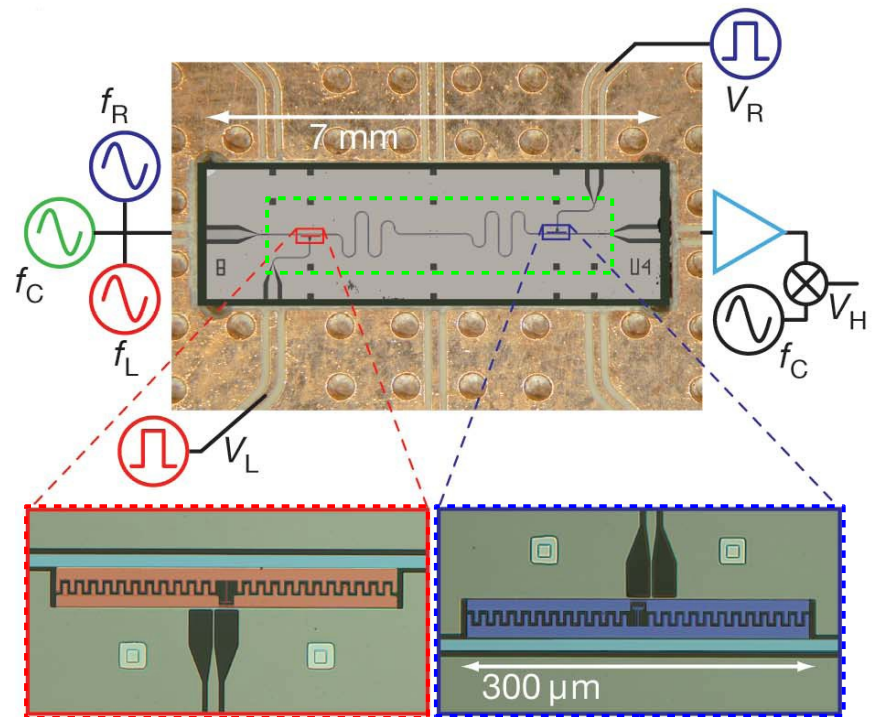 - Nb on a corundum (Al_2O_3) wafer
  - operated at 13 mK
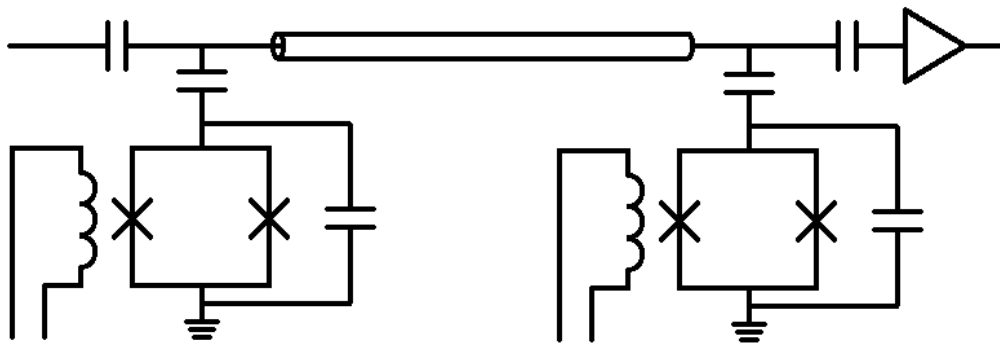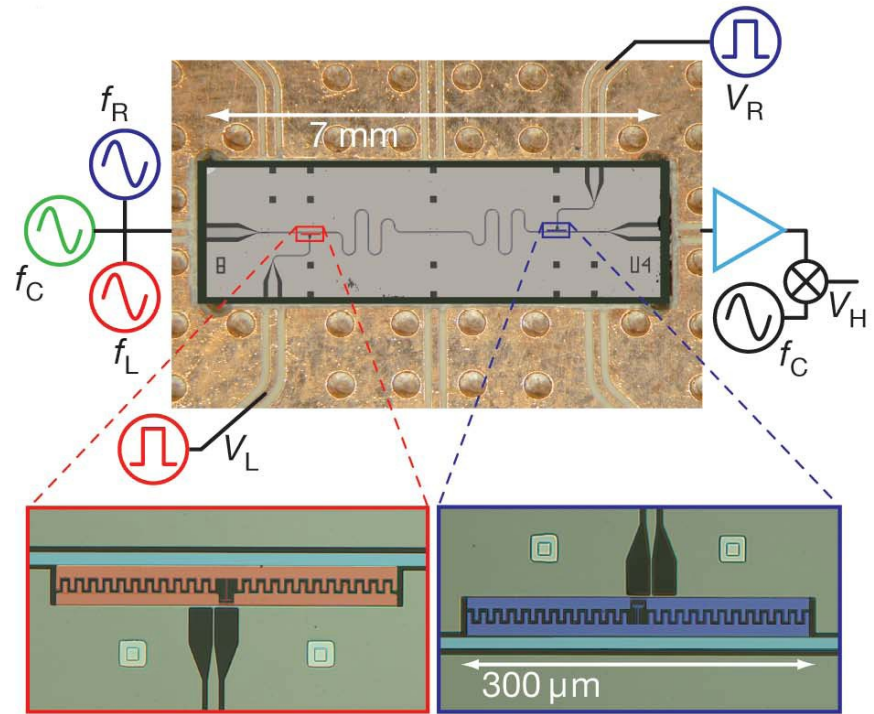
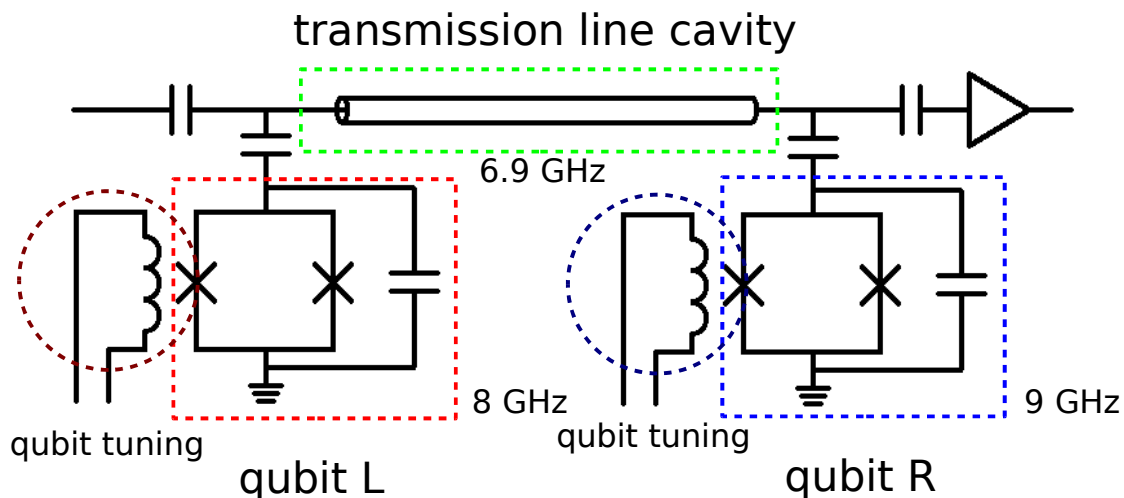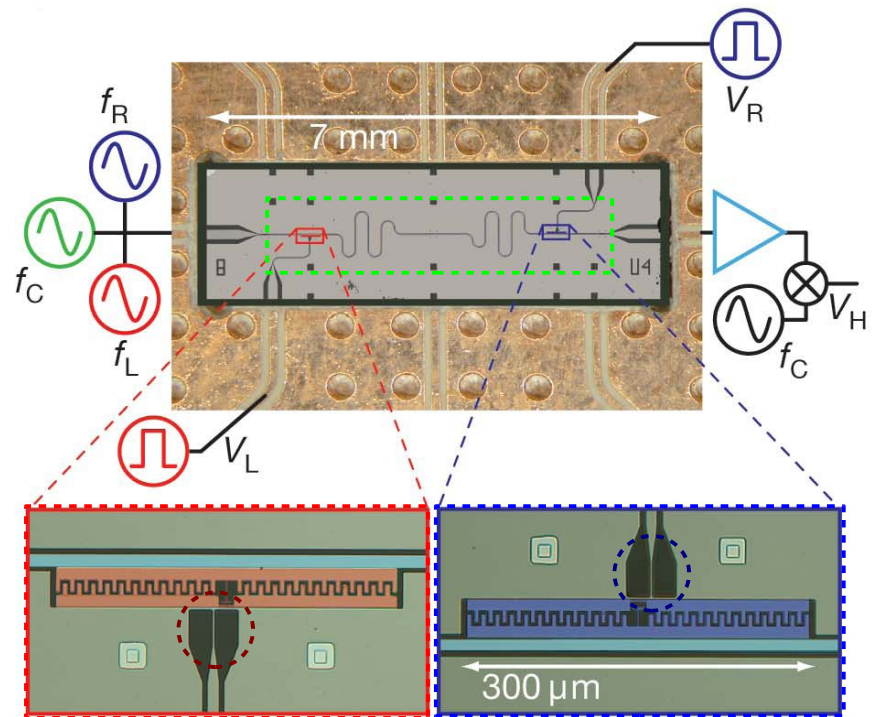# Experimental Setup

- 1D microwave (harmonic) resonator

transmission line cavity

6.9 GHz

Experimental Setup

- two __transmon qubits__

transmission line cavity

6.9 GHz

8 GHz

9 GHz
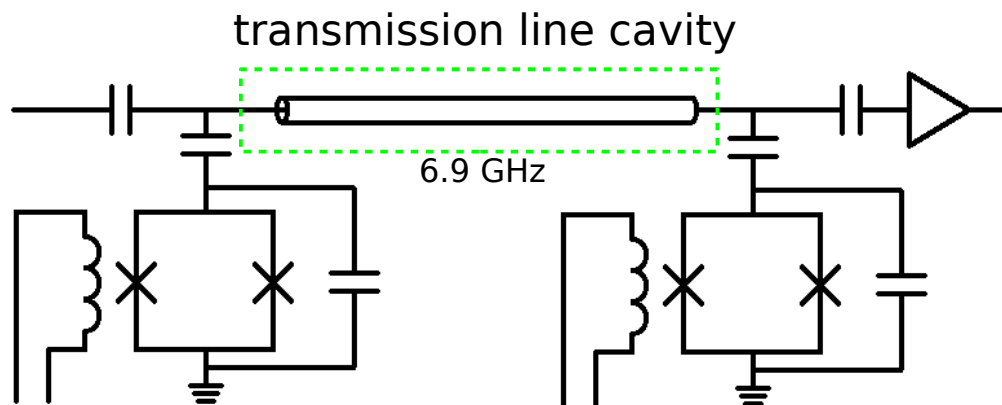
qubit L

qubit R

Experimental Setup

- two __transmon qubits__
- energy levels tunable by magnetic flux

transmission line cavity

6.9 GHz

8 GHz

9 GHz

qubit tuning

qubit tuning

qubit L

qubit R

Experimental Setup

- I/O ports of the cavity
  - for one-qubit gates
  - for qubit readout

7 mm

300 µm

in

transmission line cavity

6.9 GHz

out

8 GHz

9 GHz

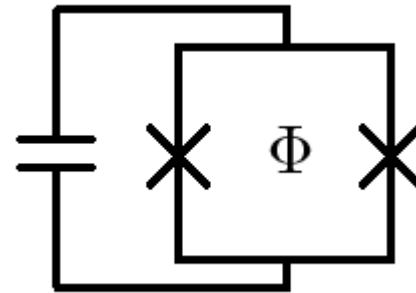qubit tuning

qubit tuning
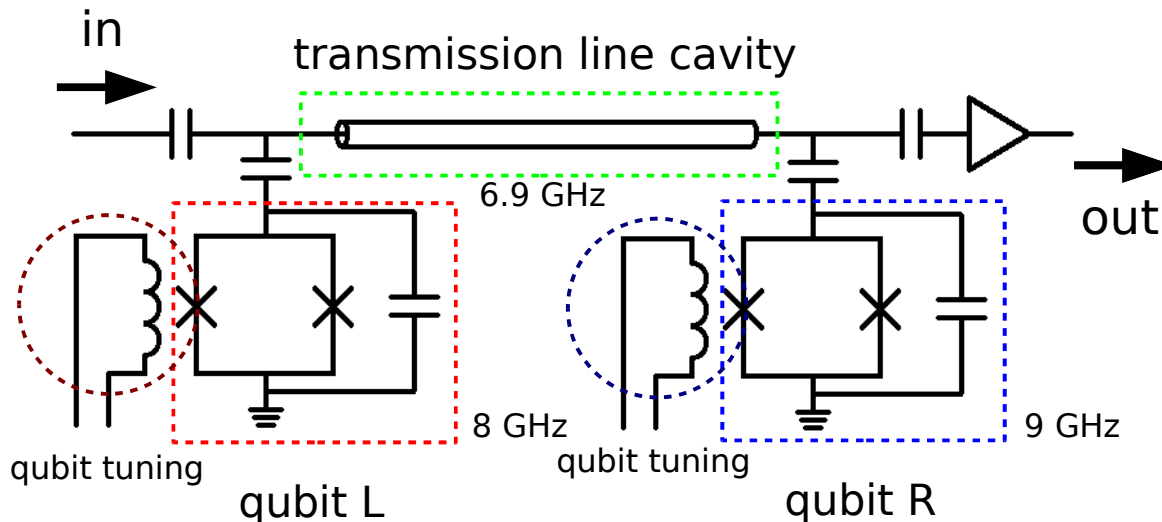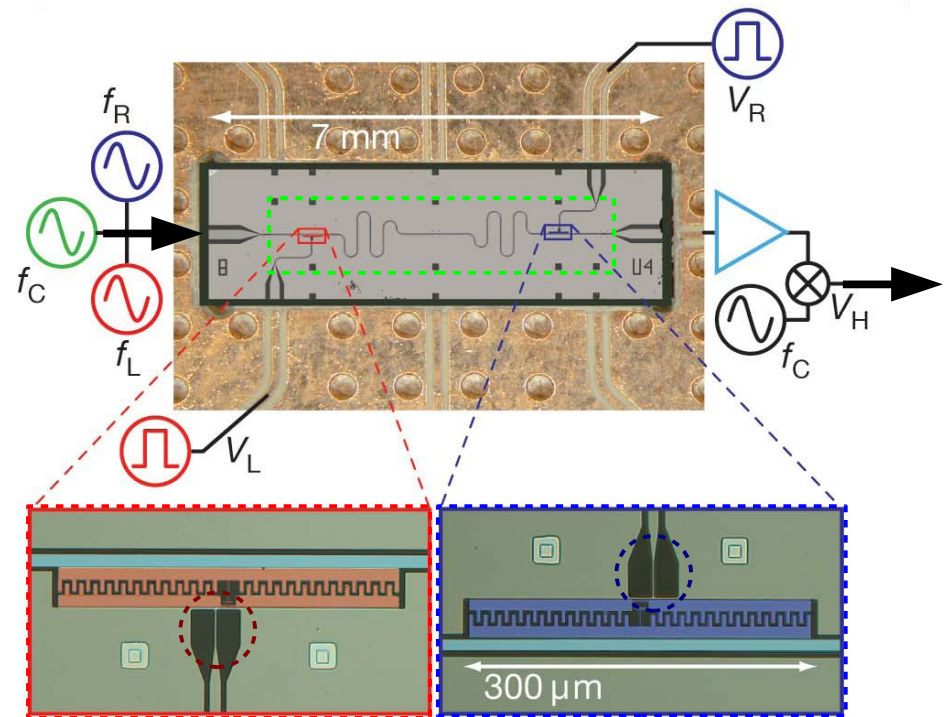
qubit L

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Transmon Qubit

- modification of a CPB
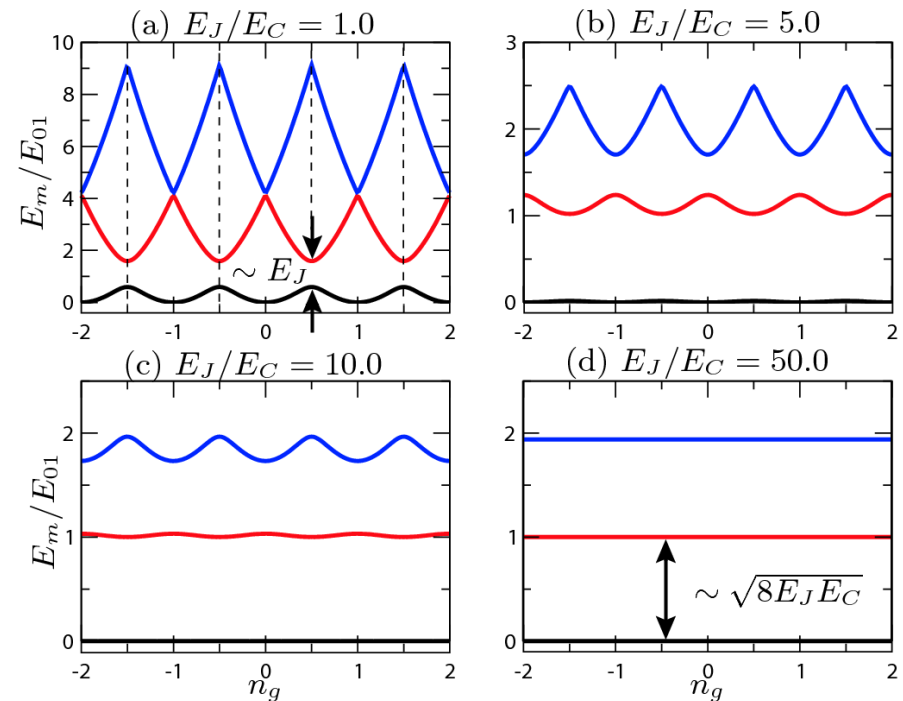
Transmon Qubit

- modification of a CPB
  - no voltage bias
  - a split Josephson junction
  - magnetic flux through the loop allows to tune Josephson energy

$$E_J = E_J^{\mathrm{max}} \left| \cos(\pi\Phi/\Phi_0) \right|$$

Transmon Qubit

- modification of a CPB
  - no voltage bias
  - a split Josephson junction
  - magnetic flux through the loop allows to tune Josephson energy

  - high ratio E_J/E_C
    → low sensitivity to charge noise

(a) $E_J/E_C = 1.0$ (b) $E_J/E_C = 5.0$
(c) $E_J/E_C = 10.0$ (d) $E_J/E_C = 50.0$

$\sim E_J$

$\sim \sqrt{8 E_J E_C}$

E_m/E_{01} n_g

J. Koch *et. al.* Phys. Rev. A **76**, 042319 (2007)

One-qubit Operations in the Superconducting Processor

- similarly to spin-1/2 qubits

  (rotations by applying transverse harmonic magnetic field)

- superconducting qubits "rotated" by a resonant microwave signal applied through the cavity
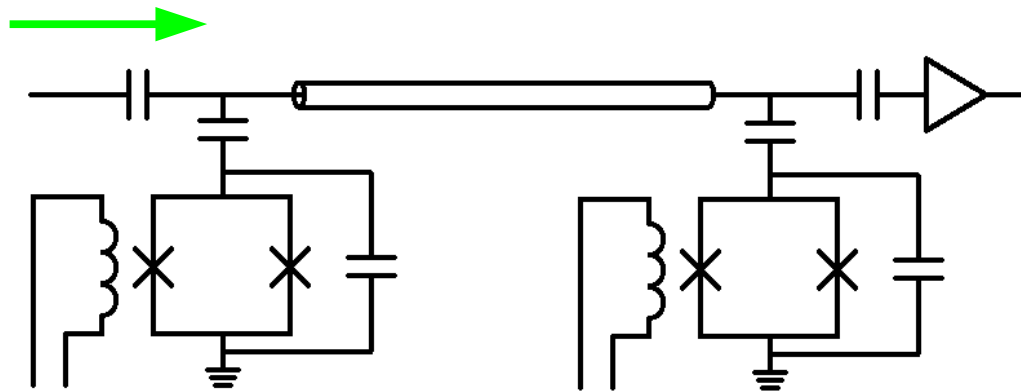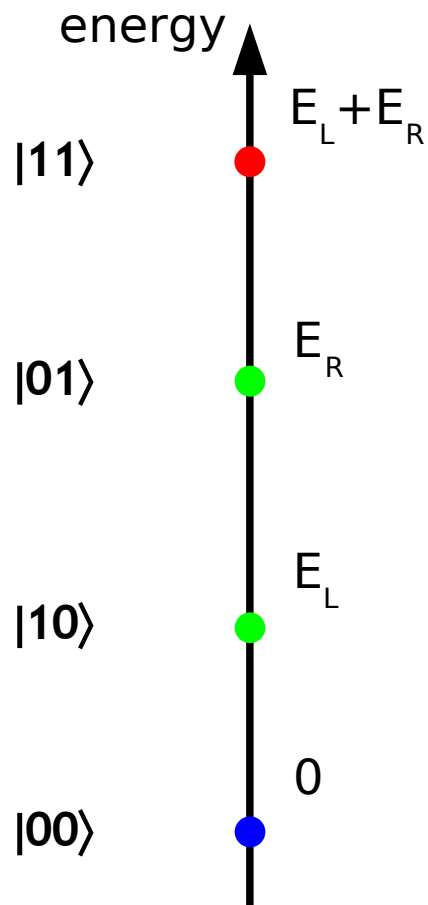
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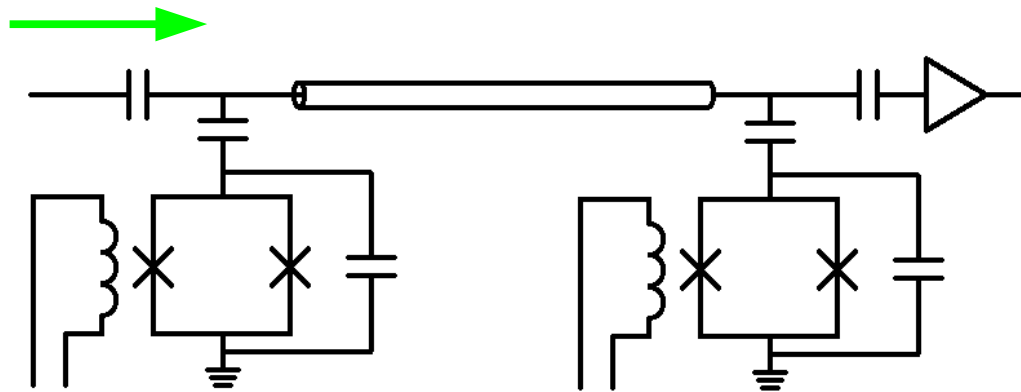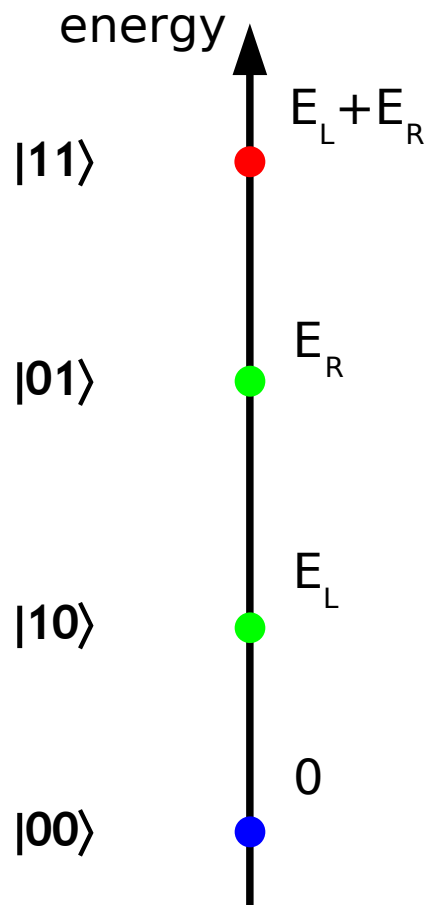

- what about two-qubit operations?

Energy spectrum of the qubits

qubits tuned to their maximum frequencies (detuned from the cavity and from each other) – effectively noninteracting

Energy spectrum of the qubits

energy

$E_L + E_R$
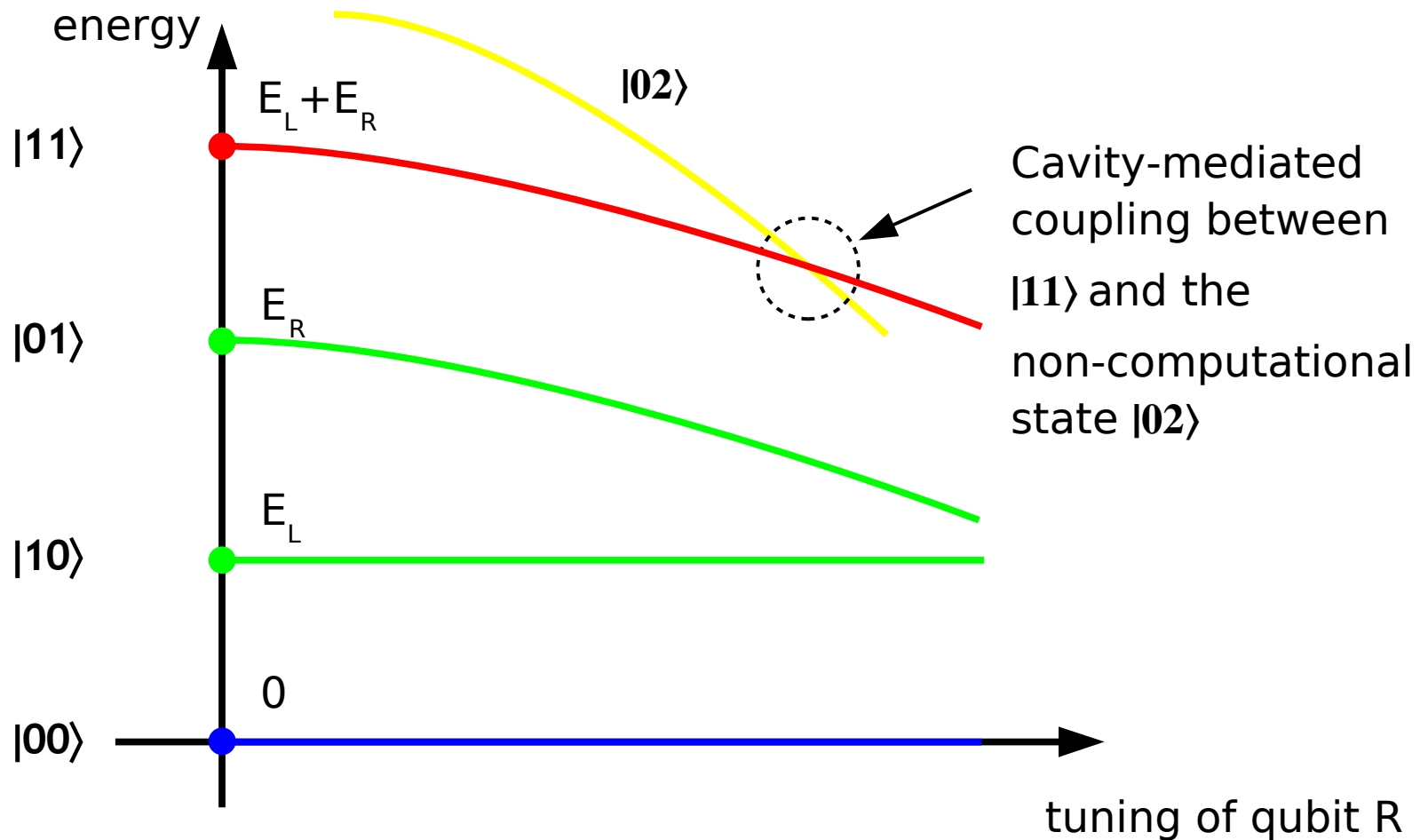
|11⟩ ●

E_R

|01⟩ ●

E_L

|10⟩ ●

0

|00⟩ ●

qubits tuned to their maximum frequencies (detuned from the cavity and from each other) – effectively noninteracting

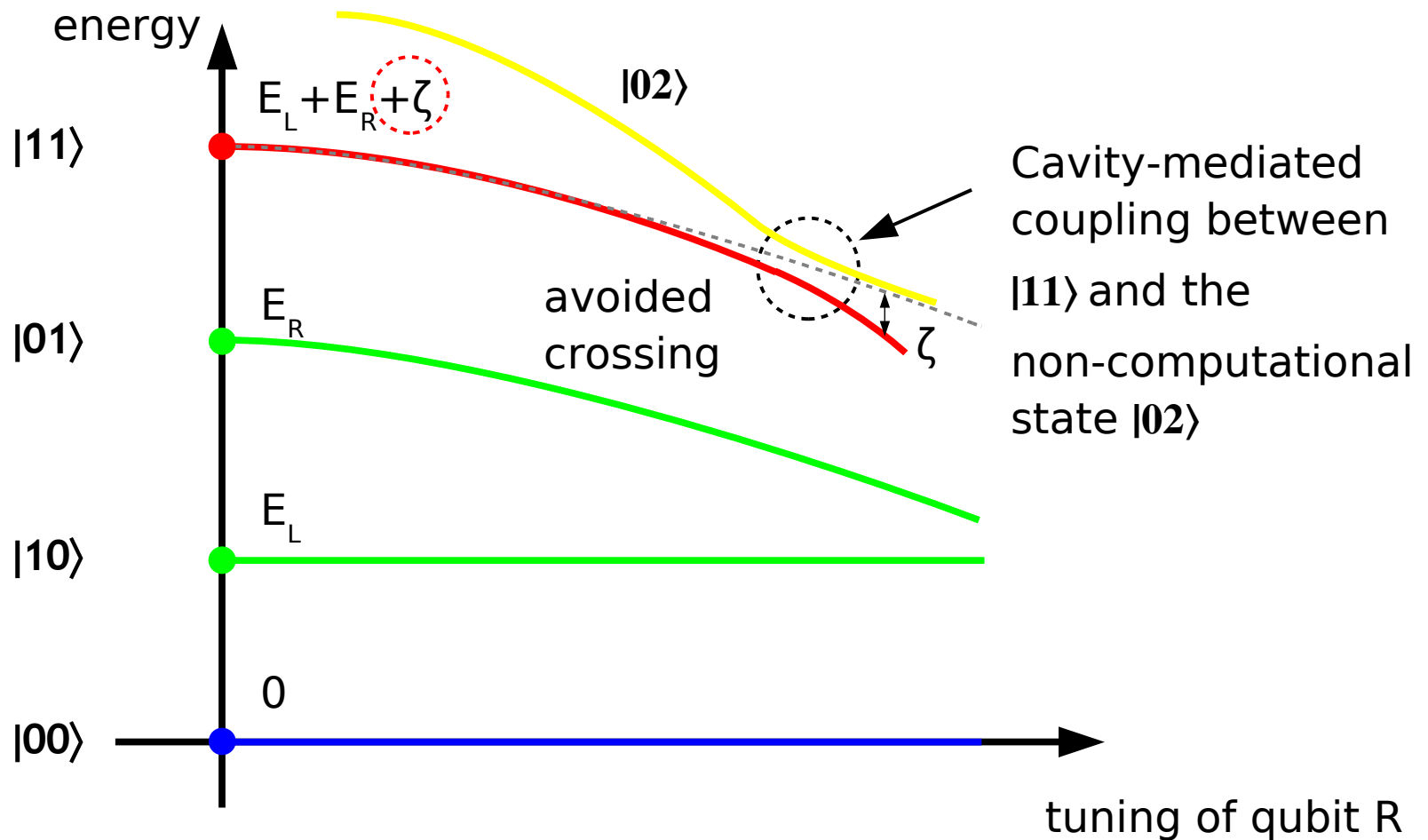system's "sweet spot" – no first order sensitivity to flux noise
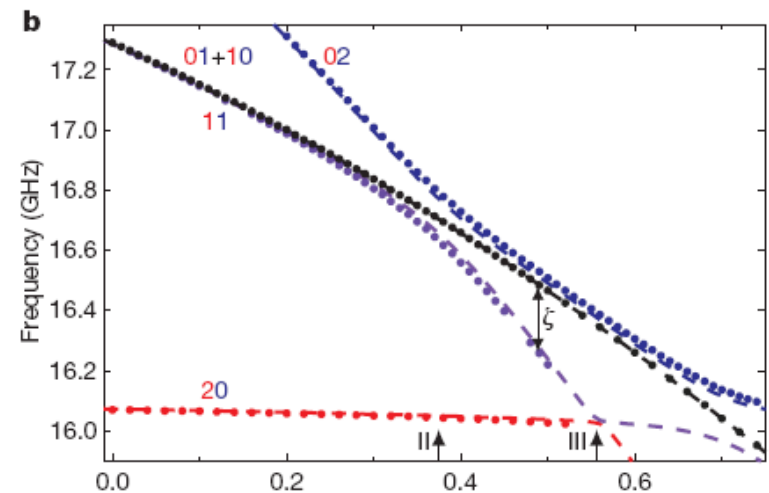
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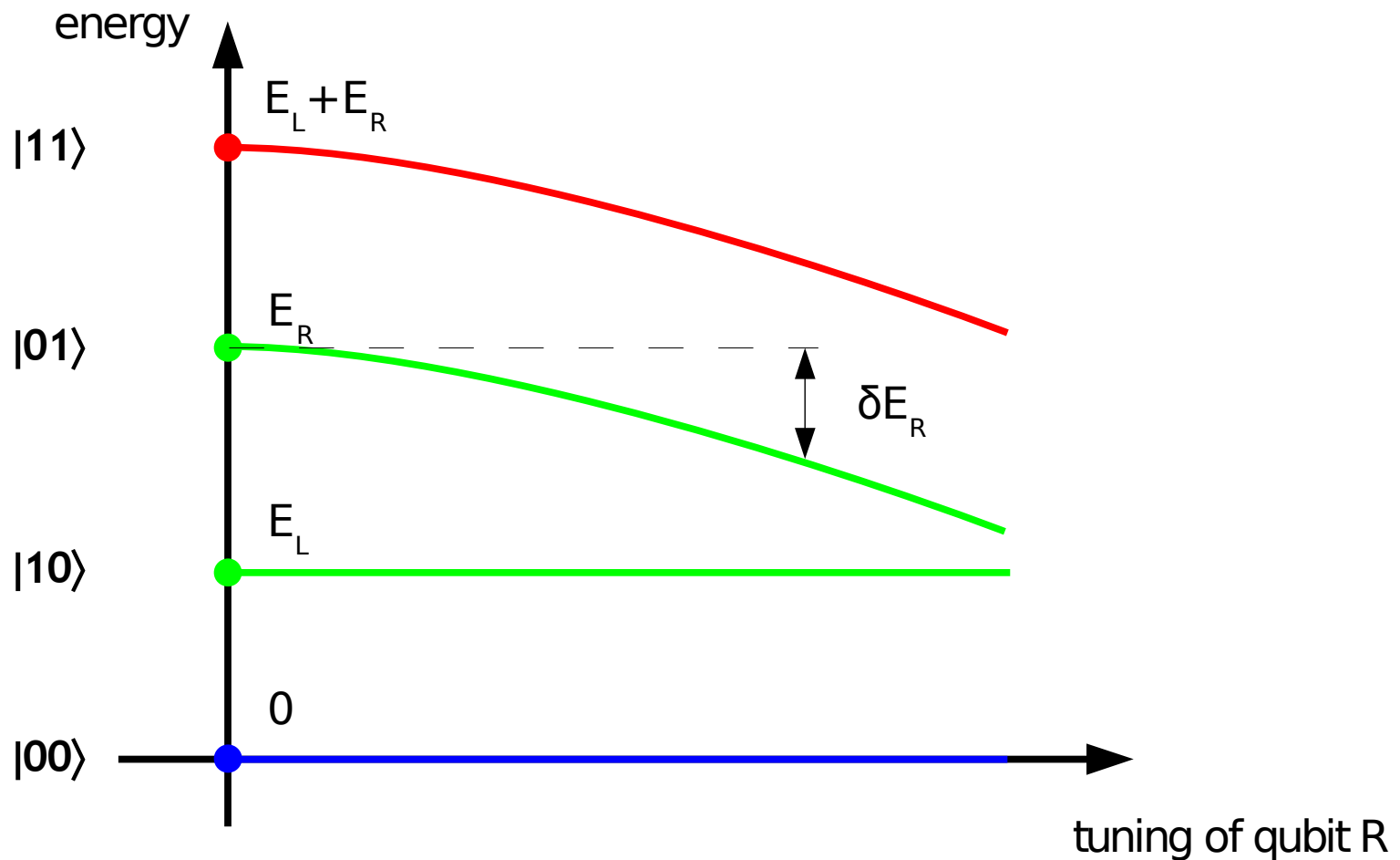
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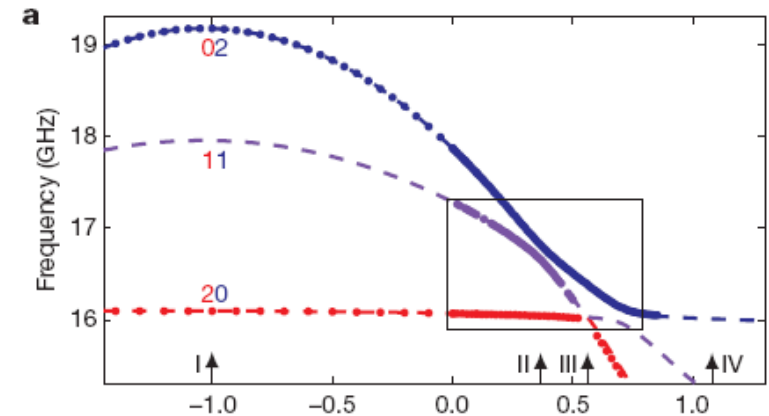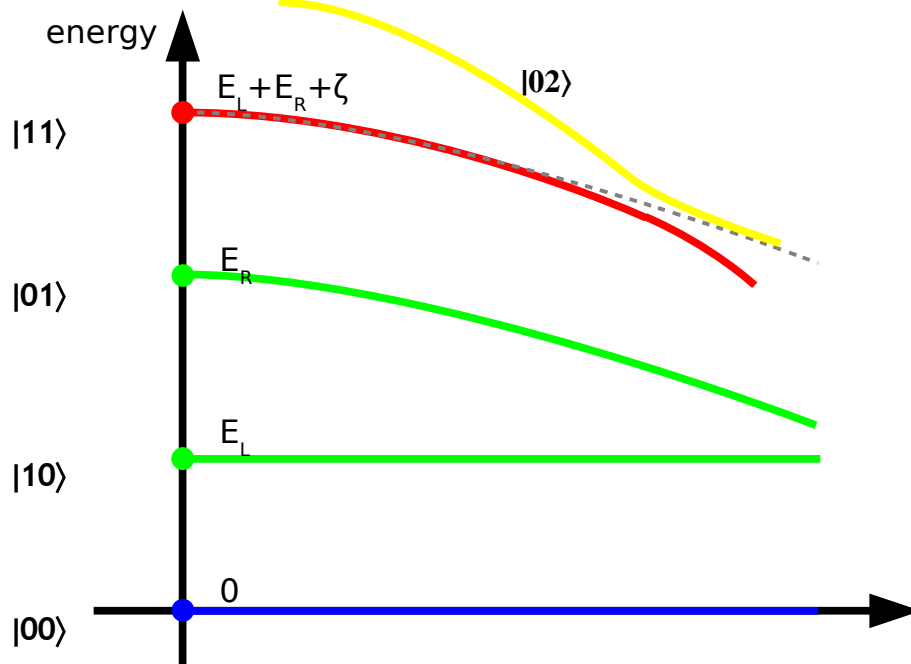
tuning of qubit R

Energy spectrum of the qubits

energy

$E_L + E_R + \zeta$

$|11\rangle$

$|02\rangle$

E_R

$|01\rangle$

E_L

$|10\rangle$

0

$|00\rangle$

tuning of qubit R

energy of qubit R varied slowly enough (adiabatically) $\Rightarrow$ system does not undergo transitions between eigenstates

eigenstates only acquire phases
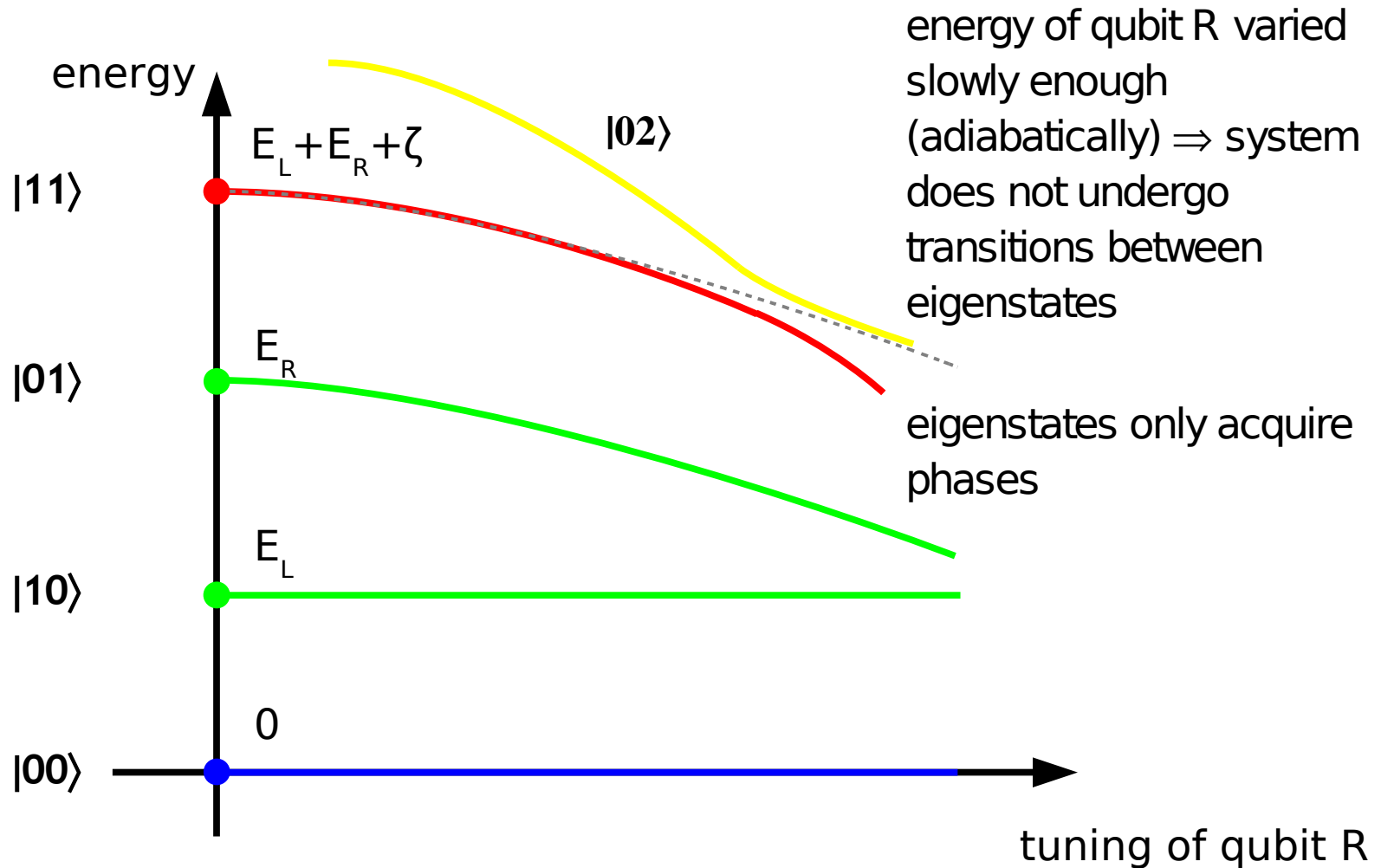
Time evolution of the system

$$|00\rangle \rightarrow |00\rangle$$

$$|10\rangle \rightarrow \exp(-i\phi_{10})|10\rangle$$

$$|01\rangle \rightarrow \exp(-i\phi_{01})|01\rangle$$

$$|11\rangle \rightarrow \exp(-i\phi_{11})|11\rangle$$

$$\phi_{10} = \frac{1}{\hbar} \int \delta E_L \, \mathrm{d}t$$

$$\phi_{01} = \frac{1}{\hbar} \int \delta E_R \, \mathrm{d}t$$

$$\phi_{11} = \frac{1}{\hbar} \int (\delta E_L + \delta E_R + \zeta) \, \mathrm{d}t$$

Time evolution of the system

$$|00\rangle \rightarrow |00\rangle \qquad\qquad \phi_{10} = \frac{1}{\hbar} \int \delta E_L \, \mathrm{d}t$$

$$|10\rangle \rightarrow \exp(-\mathrm{i}\phi_{10})|10\rangle$$

$$\phi_{01} = \frac{1}{\hbar} \int \delta E_R \, \mathrm{d}t$$

$$|01\rangle \rightarrow \exp(-\mathrm{i}\phi_{01})|01\rangle$$

$$|11\rangle \rightarrow \exp(-\mathrm{i}\phi_{11})|11\rangle \qquad \phi_{11} = \frac{1}{\hbar} \int (\delta E_L + \delta E_R + \zeta) \, \mathrm{d}t$$

Corresponding evolution operator:

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-\mathrm{i}\phi_{10}} & 0 & 0 \\ 0 & 0 & e^{-\mathrm{i}\phi_{01}} & 0 \\ 0 & 0 & 0 & e^{-\mathrm{i}\phi_{11}} \end{pmatrix}$$

Time evolution of the system

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$$\phi_{11} \neq \phi_{01} + \phi_{10}$$

U <u>cannot be factorized</u> into a tensor product of one-qubit operations

a true two-qubit operation

Creating a C-Phase gate

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-i\phi_{10}} & 0 & 0 \\ 0 & 0 & e^{-i\phi_{01}} & 0 \\ 0 & 0 & 0 & e^{-i\phi_{11}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Creating a C-Phase gate

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need to adjust the energy
levels of the qubits so that

$$\phi_{10} = \frac{1}{\hbar} \int \delta E_L \, \mathrm{d}t = 2\pi m$$

$$\phi_{01} = \frac{1}{\hbar} \int \delta E_R \, \mathrm{d}t = 2\pi n$$

$$\phi_{11} = \phi_{10} + \phi_{01} + \frac{1}{\hbar} \int \zeta \, \mathrm{d}t = 2k\pi + \pi$$

$$\Rightarrow \frac{1}{\hbar} \int \zeta \, \mathrm{d}t = -\pi$$

Creating a C-Phase gate

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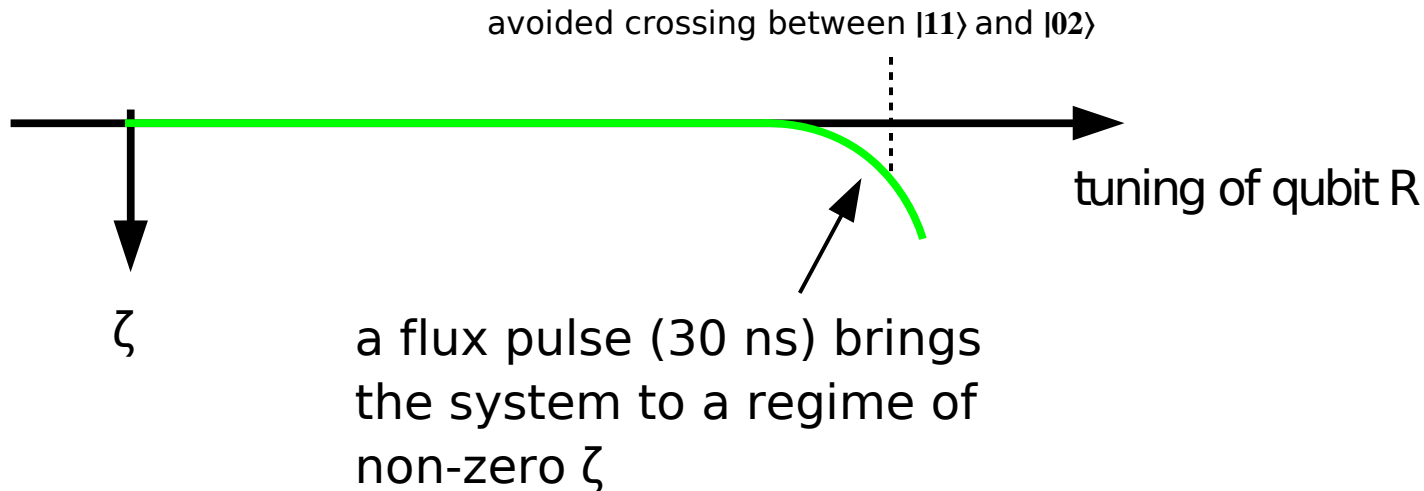$$\phi_{10} = \frac{1}{\hbar} \int \delta E_L \, dt = 2\pi m$$

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How to do this?

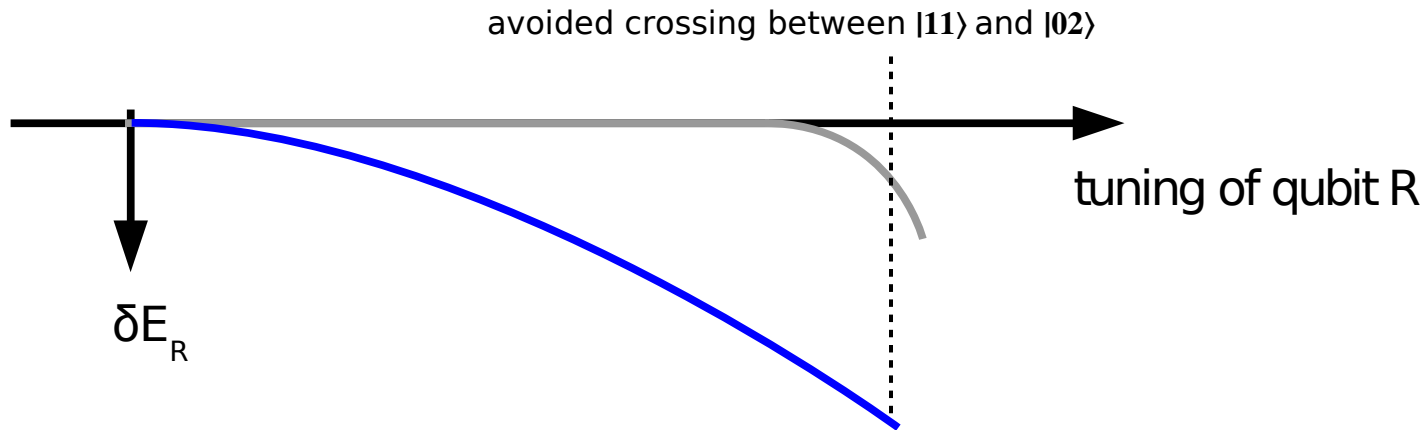$$\Rightarrow \frac{1}{\hbar} \int \zeta \, dt = -\pi$$

Creating a C-Phase gate

avoided crossing between $|11\rangle$ and $|02\rangle$

ζ

tuning of qubit R

a flux pulse (30 ns) brings the system to a regime of non-zero ζ

the exact operation point (pulse amplitude) chosen to ensure

$$\frac{1}{\hbar} \int \zeta \, \mathrm{d}t = -\pi$$

Creating a C-Phase gate

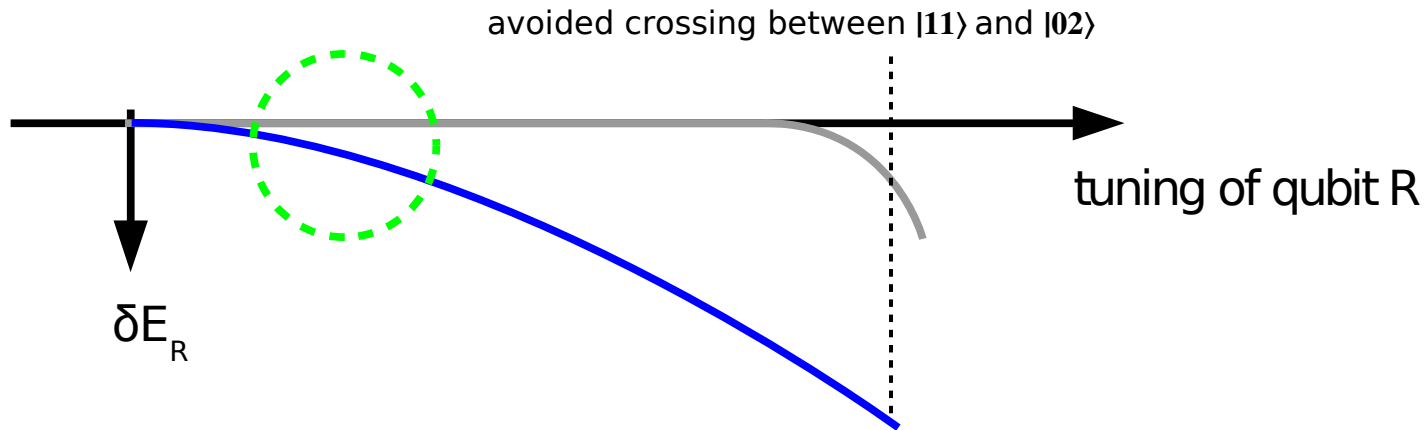avoided crossing between $|11\rangle$ and $|02\rangle$

δE_R

tuning of qubit R

system has been tuned all the way
to the avoided crossing and back

state $|01\rangle$ has acquired a large phase

$$\phi_{01} = \frac{1}{\hbar} \int \delta E_R \, \mathrm{d}t \approx -60\pi$$
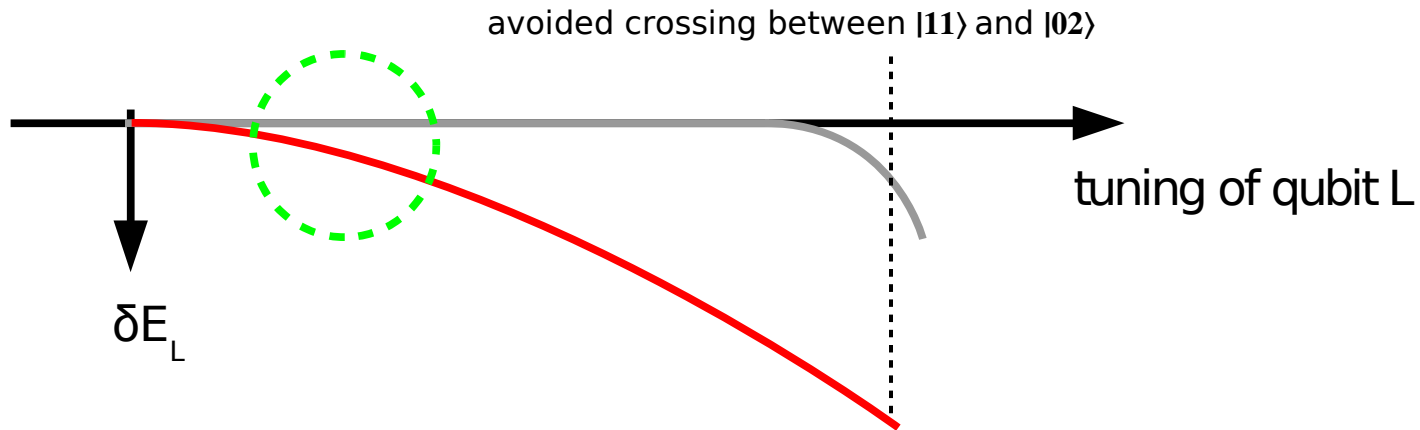
Creating a C-Phase gate

avoided crossing between $|11\rangle$ and $|02\rangle$

δE_R

tuning of qubit R

to bring Φ_{01} to the nearest multiple of 2π:

Stay in the region of non-zero δE_R (but negligible ζ) for some appropriate amount of time (adjusting edges of the pulse)

Creating a C-Phase gate

avoided crossing between $|11\rangle$ and $|02\rangle$

tuning of qubit L

δE_L

the same for the phase Φ_{10} by tuning the other qubit

Creating a C-Phase gate

just by shaping the flux pulses applied to
the qubits:

$$\phi_{10} = 2\pi m$$

$$\phi_{01} = 2\pi n$$

$$\phi_{11} = 2\pi k + \pi$$

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

What can we do once we have the C-Phase gate?

Let's do some simple math first.

- Do $R_y^{\frac{\pi}{2}}$ to both qubits initially in state |0>, then apply cU00 and then do $R_y^{\frac{\pi}{2}}$ only to the "Left" qubit.

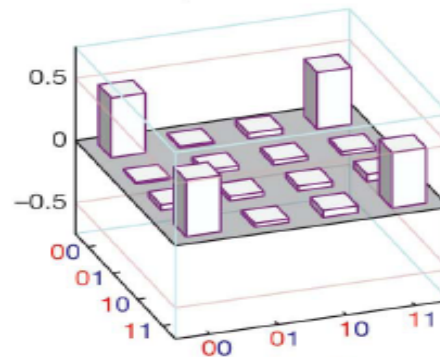| Initial state	|0>⊗ |0>
$R_y^{\frac{\pi}{2}} \quad R_y^{\frac{\pi}{2}}$	$(|0>+|1>)/\sqrt{2} \quad\otimes\quad (|0>+|1>)/\sqrt{2}$
	|0,0> + |0,1> + |1,0> + |1,1>
cU00 gate	-|0,0> + |0,1> + |1,0> + |1,1>
$R_y^{\frac{\pi}{2}}$ Identity	-|(0+1)/$\sqrt{2}$,0> +|(0+1)/$\sqrt{2}$,1> + |(0-1)/$\sqrt{2}$,0> + |(0-1)/$\sqrt{2}$,1>
	-|0,0> - |1,0> + |0,1> + |1,1> + |0,0> - |1,0> + |0,1> - |1,1>
	(|0,1> - |1,0>)/$\sqrt{2}$ - one of Bell's states

- Note: $cU_{00}\,|\,l,r>=(-1)^{\delta_{ol}\delta_{or}}\,|\,l,r>$

Experimental results with entanglement.

- Fidelities –
- how close
- to expected:
- ~ 87%
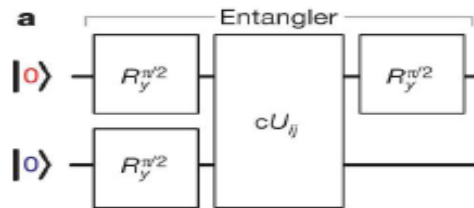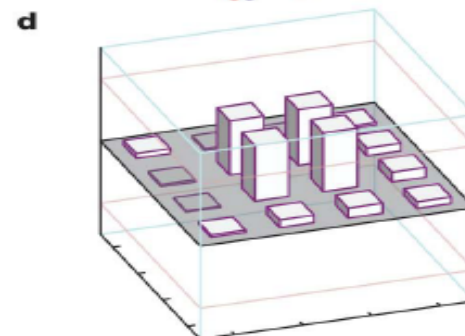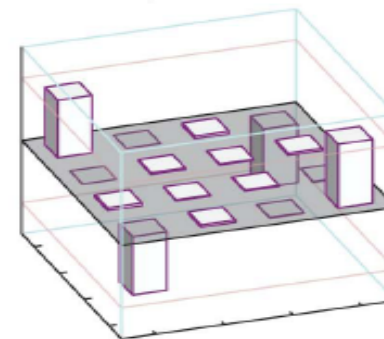
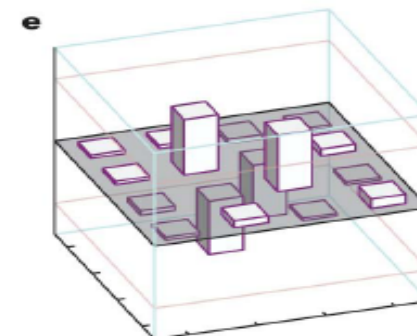# Grover search algorithm.
Outline

- Problem statement.
- See how it can be solved with quantum processor.
- Where does the gain in performance come from?
- How it was realised @ 2 qubit system.

Marek Pechal, Kiryl Pakrouski

Problem statement.

- Given:

1. n-bit input register.   Ex: (--)

2. All possible $(N = 2^n)$ numbers that can be stored in such a register – {x}.   Ex: {00,01,10,11}

3. 1-bit output register.   Ex: (-)

4. Function *f(x)* which sets output bit in case x is "the one":

   f(x) = 1 for *x=a* ← "the one"

   f(x) = 0 otherwise

Find:

"The one" - *xi* - among all {x} for which *f(xi) = 1*

Marek Pechal, Kiryl Pakrouski

Analogy to database search

- Reformulation:

  Given a phonebook find the person whose number is 1234.

- Classically one has to apply number comparison function at least N/2 times to rows of phonebook to find the person with 50% probability

- Grover algorithm needs only $\dfrac{\pi}{4}\sqrt{N}$ trials

Name	Phone #
Bob	100500
Alice	1234
Kiryl	101500
Marek	102500
……	……..

Quantum computation hints & tricks.

- **Hint #1**: Represent "comparison" function f with unitary operator:

$$U_f(|input>_n, |output>_1) = |input>_n |output \otimes f(input)>_1$$

- Flip output bit iff f detects "the one".

- **Hint #2**: output in the form:

$$\frac{1}{\sqrt{2}}(|0> - |1>)$$

property – this will change the sign

when the function f detects "the one" and flips 0 to 1.

- We've constructed "comparison" operator which just flips the sign of input when the input is "the one":

$$V_a|x> = (-1)^{f_a(x)}|x>$$

- Eureka! We could use our brand-new cGate here!

Where does the gain in performance come from?

- The operator V is linear. We can apply it to the superposition of all possible inputs! This corresponds to quantum parallelism!

- Note that when applied to the superposition state Ψ it will only flip the components along "the one" vector **a**. Components perpendicular to **a** stay unchanged.

- Hence the operator can be written as:

$$V_a = I - 2P_a = I - 2|a><a|$$
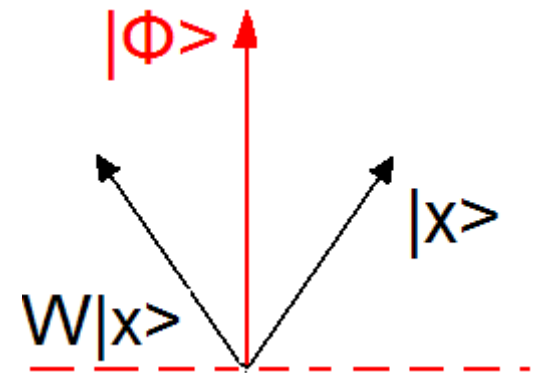
- Graphically:
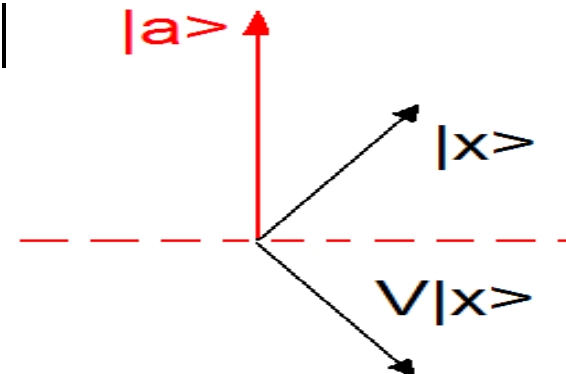
Need to construct one more operator.

- Consider full superposition of all possible input states

- $$\phi = \frac{1}{2^{n/2}} \sum_{x=0}^{2^n-1} | x >_n \qquad Note: < a | \phi > = \frac{1}{2^{n/2}} = \sin(\theta) = \sin(a_\perp \wedge \phi) \approx a_\perp \wedge \phi$$

- Construct operator W that

1. Flips the part of input that is $\perp \, to \, \phi$
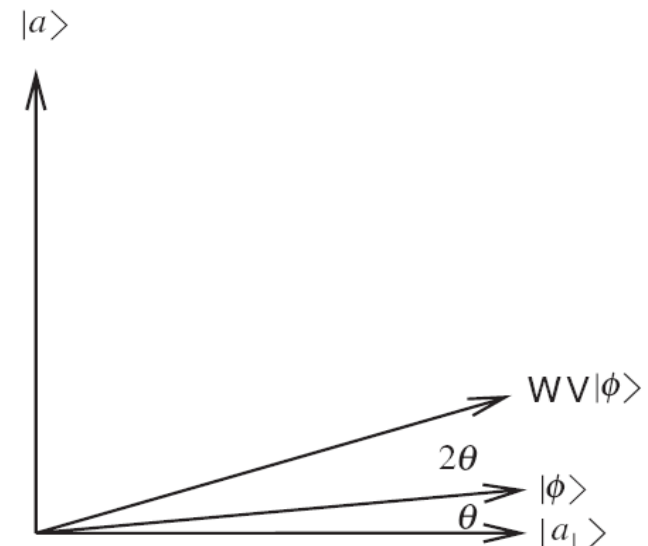2. Retains the part parallel to ϕ

$$W = 2 | \phi >< \phi | - I$$

Algorithm.

- Pretty straightforward:
- Apply operator WV to state ϕ $\frac{\pi}{4} 2^{n/2}$ times.
- How does that work?
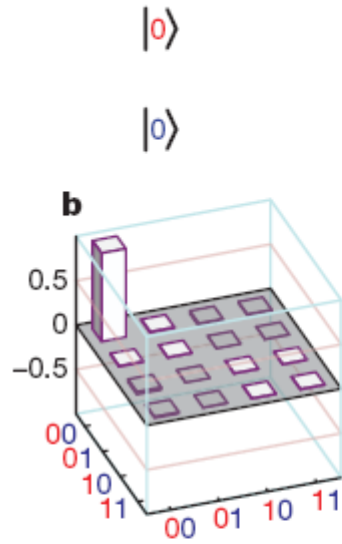- 1 iteration of Grover algorithm rotates state by angle 2θ towards unknown state |a>.
- In case of 2 qubits:

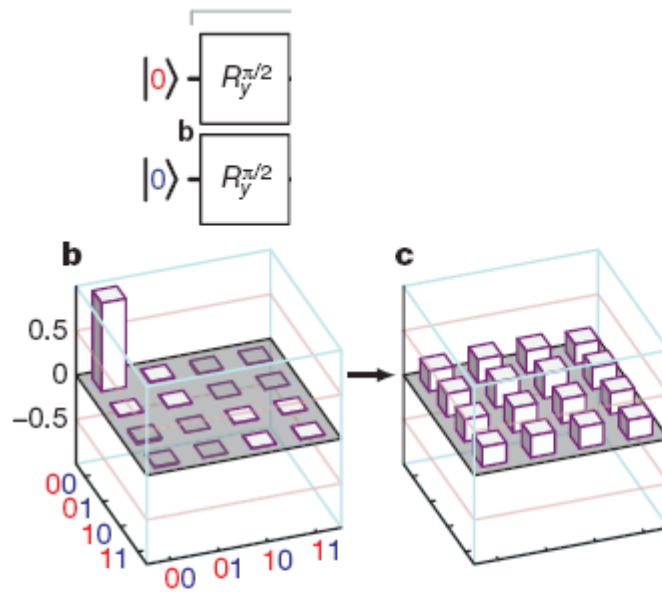$$\sin\theta = \frac{1}{2^{n/2}} = \frac{1}{2^{2/2}} = \frac{1}{2} => \theta = 30°$$

- So 1 iteration (rotation by 60) yields the exact solution. (as opposed to 3 trials classically)

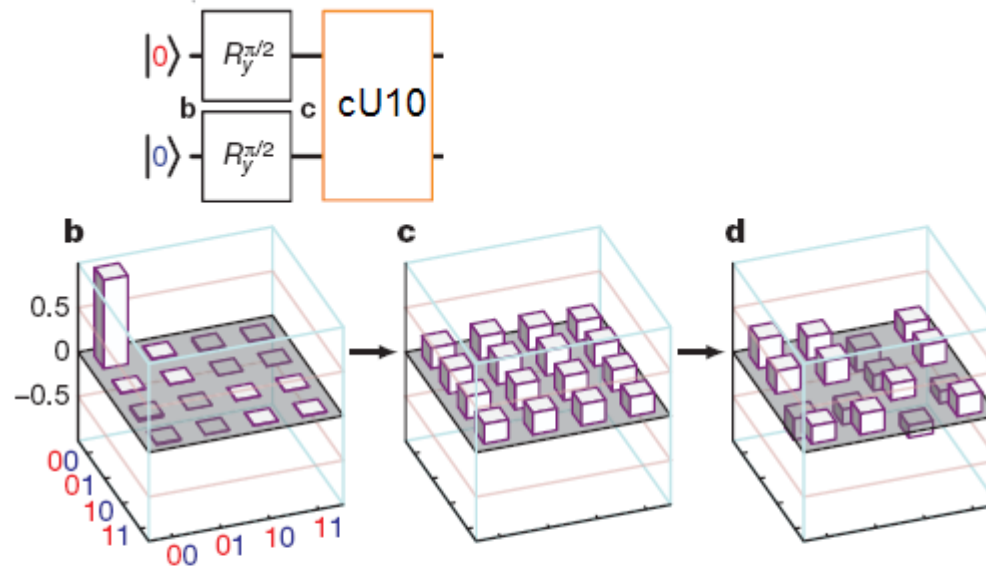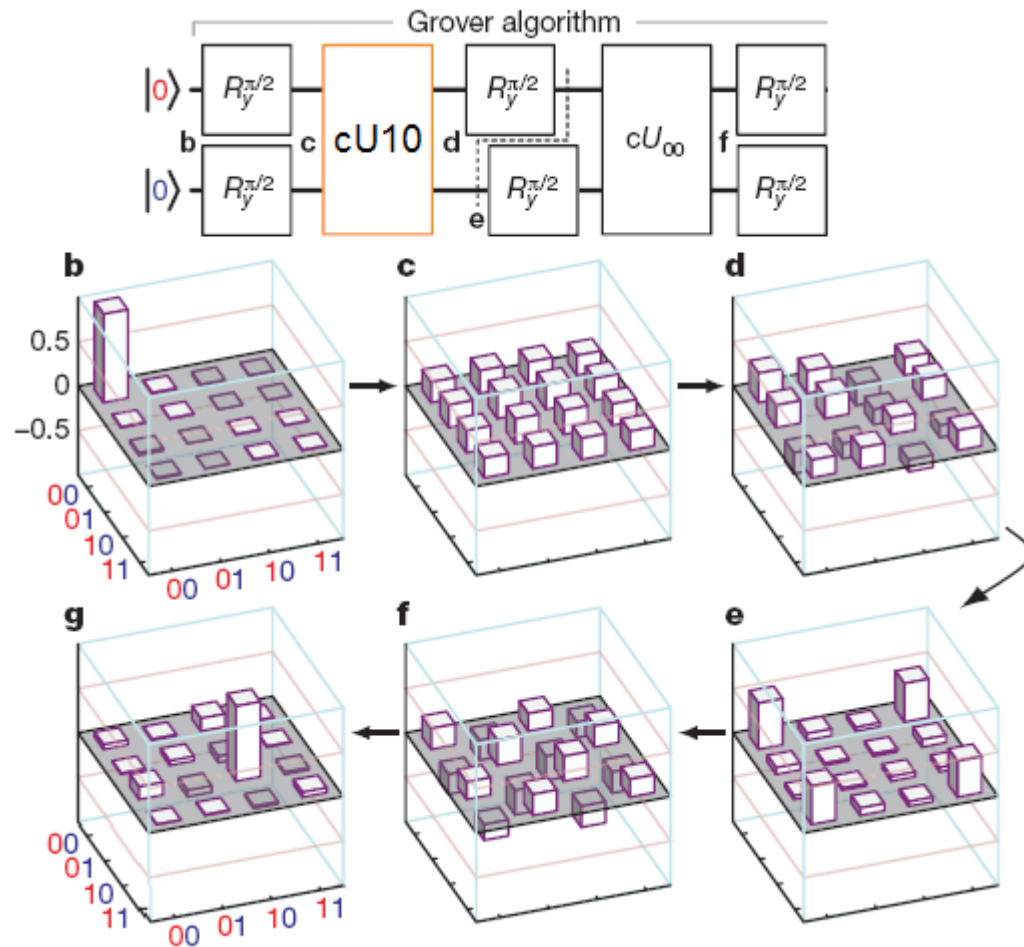# Experimental results for 2-qubit system.

$|0\rangle$

$|0\rangle$

Experimental results for 2-qubit system.

Experimental results for 2-qubit system.

Experimental results for 2-qubit system.

How do you measure the state?
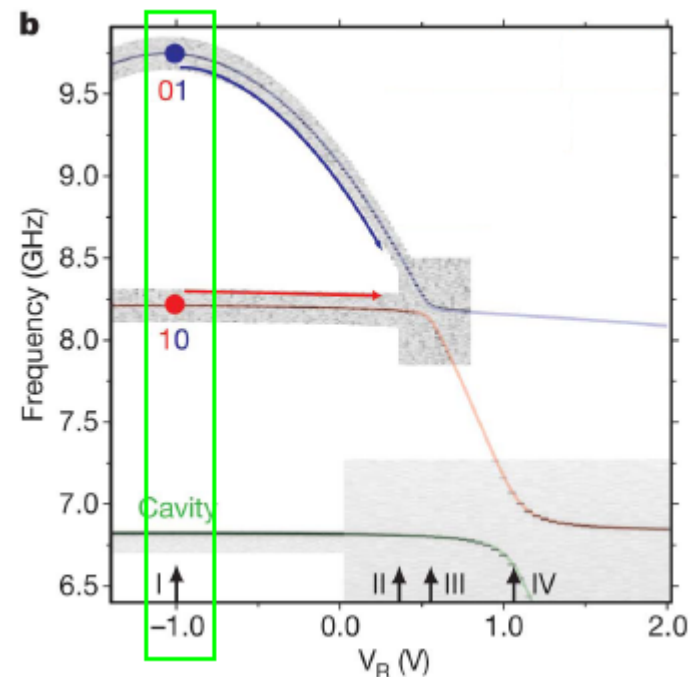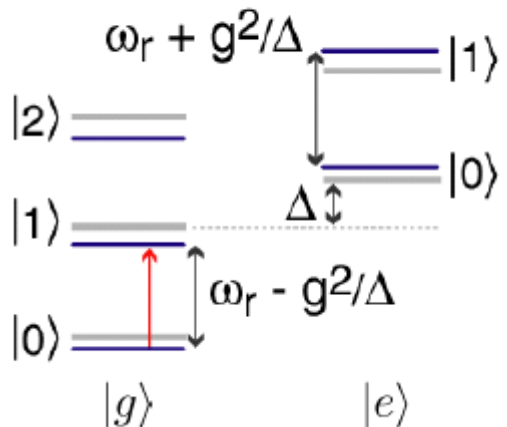# Joint dispersive readout.

- How 2 qubits interact with cavity?

- What you actually measure?

- How to extract information out of your measurements?

Cavity interaction with 1 qubit.

- Work in a strong-dispersive regime – when qubits are far detuned from each other and from the cavity.

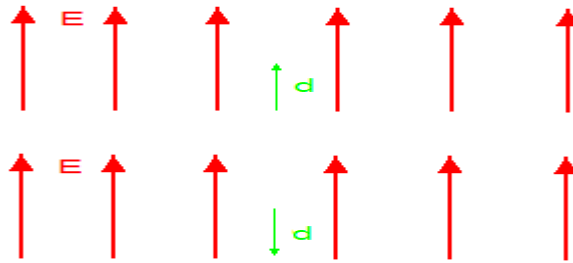- Approximate Hamiltonian diagonalisation:

$$H \approx \hbar \left(\omega_r + \frac{g^2}{\Delta}\sigma_z \right) a^\dagger a + \frac{1}{2}\hbar \left(\omega_a + \frac{g^2}{\Delta} \right) \sigma_z$$

Recall from the lecture: cavity level shift in case of 1 qubit

$\omega_r + g^2/\Delta$ ——— $|1\rangle$

$|2\rangle$ ———

——— $|0\rangle$

$|1\rangle$ ——— $\Delta \updownarrow$

$\omega_r - g^2/\Delta$

$|0\rangle$ ———
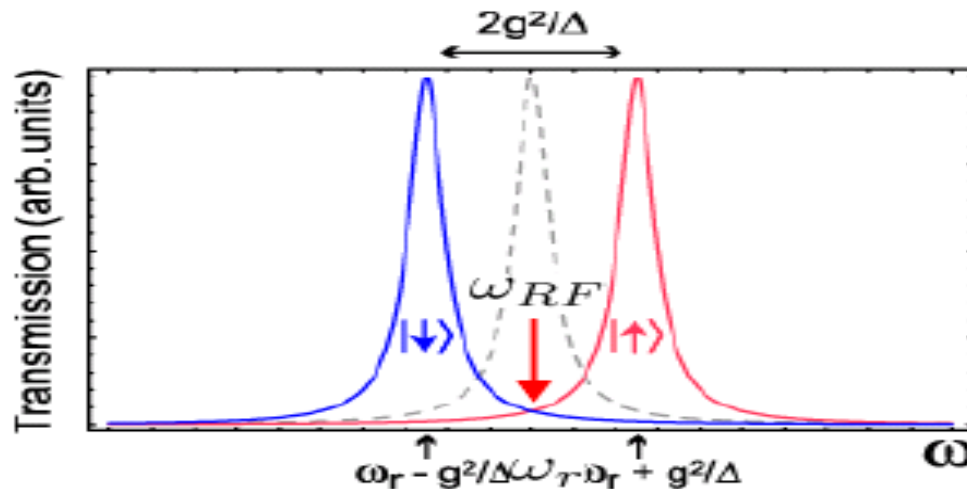
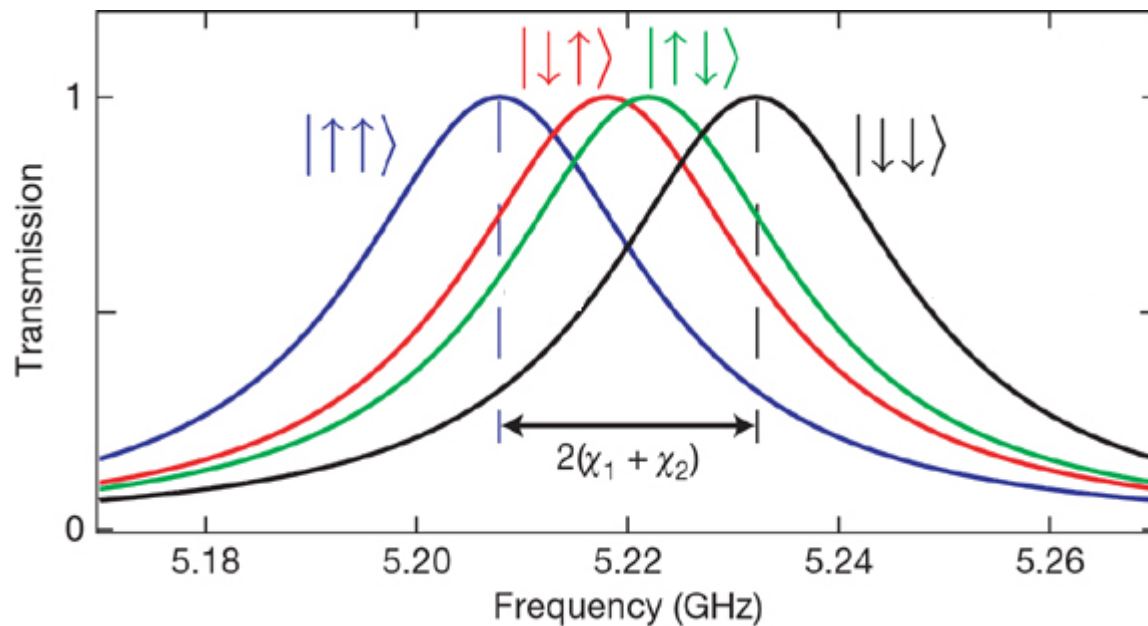$|g\rangle$ $|e\rangle$

Cavity-qubits interaction.

- Can also be thought of by analogy with EM-field-dipole interaction:

- Transmission of a cavity with one cubit in it: A. Blais, R.-S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, *PRA* **69, 062320 (2004)**
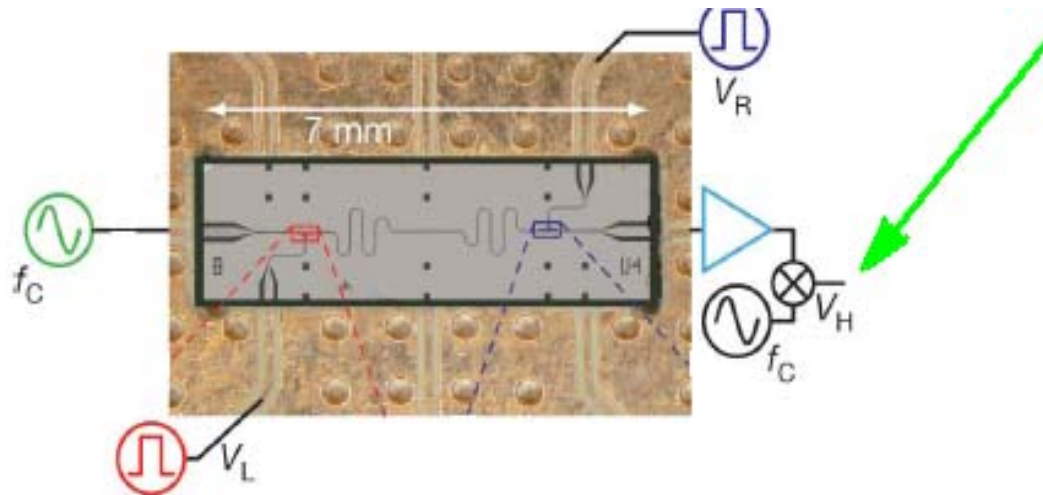
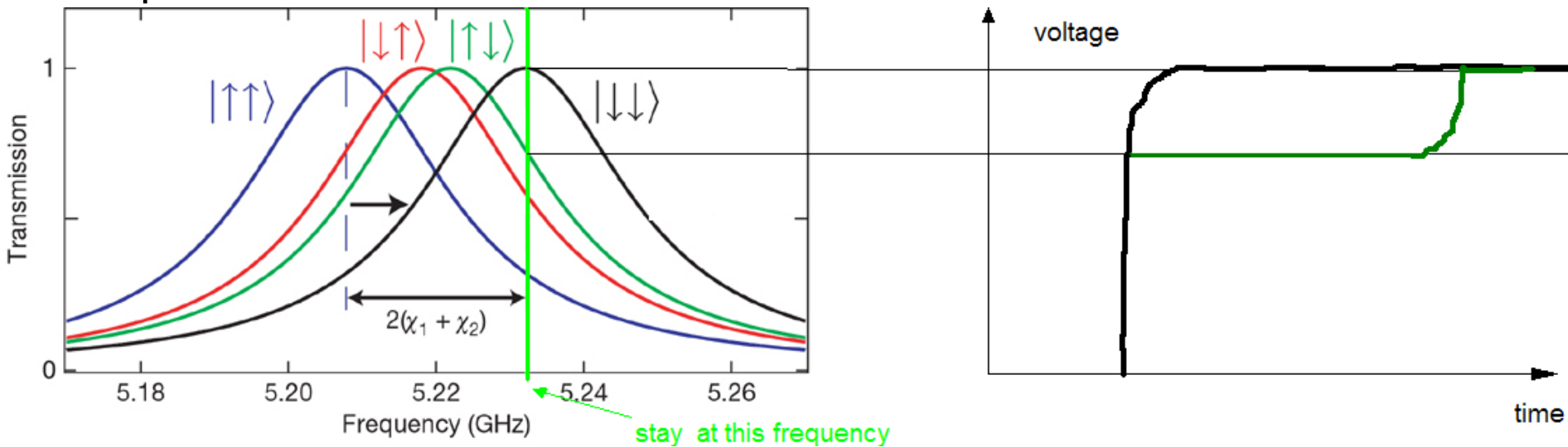# Transmission of a cavity with 2 qubits.

What you actually measure?

- Stay at some frequency around the resonance of your cavity.

- Measure homodyne voltage:

What you actually see?

- Consider 2 different (|e> and |g>) states of "Left" qubit. Let the "Right" qubit remain in the same state.

- Note that voltage above is shown for 2 different measurements
- By playing around with such measurements one can say what value corresponds to which system's state.
- You can also extract some information about decay rate.
- Coupling superconducting qubits via a cavity bus
- J. Majer, et al Nature 449, 443-447(27 September 2007)

How to get nice pictures of density matrix?

- Formally measurement for a quantum bus with 2 qubits can be represented by following operator:

$$M = \beta_1 \sigma_z^{\mathrm{L}} + \beta_2 \sigma_z^{\mathrm{R}} + \beta_{12} \sigma_z^{\mathrm{L}} \otimes \sigma_z^{\mathrm{R}}$$

- Filipp, S. *et al. Two-qubit state tomography using a joint*
- dispersive readout. *Phys. Rev. Lett.* **102, 200402 (2009).**

- One measures σ^z operator (coefficients should be calibrated) but density matrix has 16 components (trace=1).

- Do set of 15 rotation combinations:

$$\{\mathrm{I}, R_x^\pi, R_x^{\frac{\pi}{2}}, R_y^{\frac{\pi}{2}}\}_{left\_qubit} \otimes \{\mathrm{I}, R_x^\pi, R_x^{\frac{\pi}{2}}, R_y^{\frac{\pi}{2}}\}_{right\_qubit} - \{R_x^\pi \otimes R_x^\pi\}$$

- Repeat 450.000 times{

 - Prepare state and do rotations

  - Measure

Can you just use average values of 15 measurement operators?

- You can. But the resulting matrix might not be hermitian and the trace might not be 1.

- You can do Maximum likelihood by looking to a matrix that is very similar with what you get but also satisfies the above criteria.

- Find matrix ρ with the components that minimize the discrepancies between the actual matrix you get and your "fitting matrix" :

$$\mathcal{L} = \sum_{i=1}^{16} (m_i - \text{Tr}[M_i \rho])^2$$

Plot your results and enjoy high fidelities.

- Note: Fidelity – a measure of how much your measured state overlaps with the "ideal" state you expected. (85% for final state of the Grover algorithm below)
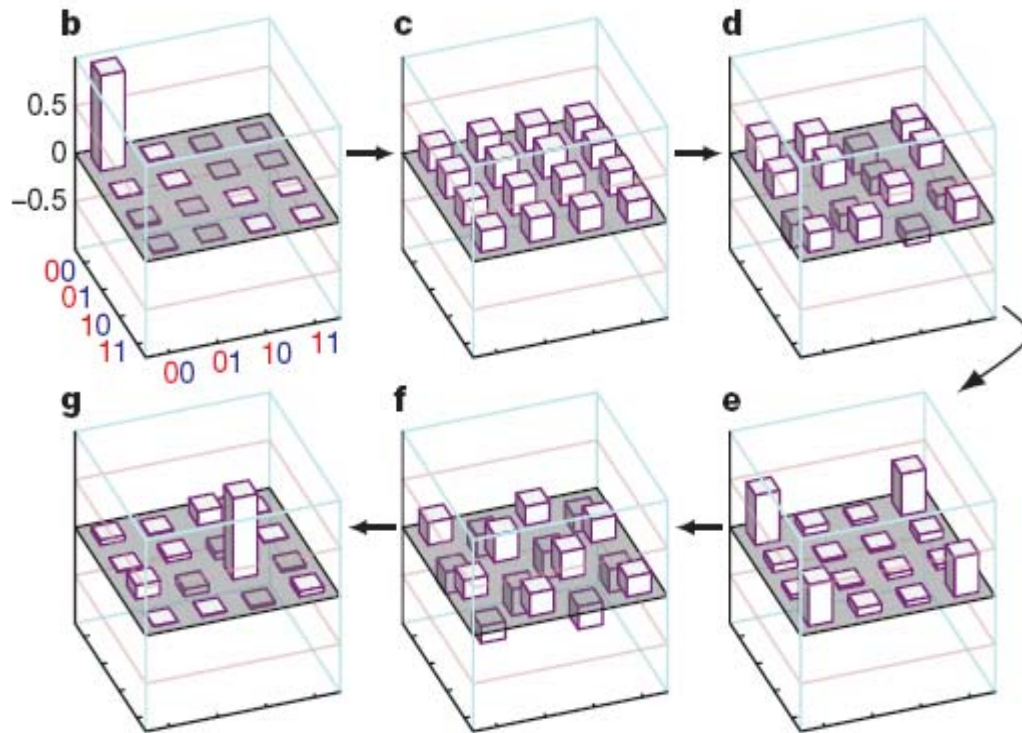
DiVincenzo criteria, summary.

1. A scalable physical system with well-characterized qubits.

(*Authors claim that the system can be immediately expanded to several qubits. One can also think of making some 3D array of 1D cavities*)

2. The ability to initialize the state of the qubits.

3. Coherence times Td, much longer than the gate-operation time Tg. *(Td ~1µs, Tg_1q~5ns, Tg_2q~30ns)*

4. A universal set of quantum gates.

5. A qubit-specific measurement capability.

6. The ability to interconvert stationary and mobile (or flying) qubits.

7. The ability to faithfully transmit flying qubits between specified locations.