The Josephson Junction as a Non-Linear Inductor

induction law $V = -L\dot{I}$

Josephson equations $I = I_0 \sin\delta$ $\boxed{dc}$

$V = \frac{\Phi_0}{2\pi}\dot{\delta}$ $\boxed{ac}$

with $\dot{I} = I_0 \cos\delta\,\dot{\delta}$

follows $\boxed{V = \frac{\Phi_0}{2\pi I_0}\frac{1}{\cos\delta}\dot{I}} = L_J \dot{I}$

Josephson inductance $L_J = L_{J0}\frac{1}{\cos\delta} \rightarrow$ non-linearity

$L_{J0} = \frac{\Phi_0}{2\pi I_0}$ specific Josephson inductance

<u>Note</u>: Phase difference δ in Josephson junction can be regarded as normalized magnetic flux

$$\delta = 2\pi\frac{\Phi}{\Phi_0}$$

Josephson Inductance and Josephson Energy

- Josephson energy

$$E_J = \int V I \, dt$$

$$= \int \frac{\Phi_0}{2\pi} \dot{\delta} \, I_0 \sin\delta \, dt$$

$$= \frac{\Phi_0 I_0}{2\pi} \cos\delta$$

$$= E_{J0} \cos\delta \qquad \text{with } E_{J0} = \frac{\Phi_0 I_0}{2\pi}$$

- typical parameters:

$$I_0 = 100 \text{ mA}$$

$$\Rightarrow L_{J0} = \frac{\Phi_0}{2\pi I_0} \approx 3 \text{ mH} \qquad (\sim 3 \text{ mm of wire})$$

$$\Rightarrow E_{J0} = \frac{\Phi_0 I_0}{2\pi} \approx 50 \text{ GHz}$$