Building a Quantum Information Processor using Superconducting Circuits

The DiVincenzo Criteria

for Implementing a quantum computer in the standard (circuit approach) to quantum information processing (QIP):

#1. A **scalable** physical system with well-characterized qubits.
#2. The ability to **initialize** the state of the qubits.
#3. **Long (relative) decoherence** times, much longer than the gate-operation time.
#4. A **universal set** of quantum gates.
#5. A qubit-specific **measurement** capability.

plus two criteria requiring the possibility to transmit information:

#6. The ability to **interconvert** stationary and mobile (or flying) qubits.
#7. The ability to faithfully **transmit** flying qubits between specified locations.
The challenge:

- Quantum information processing requires excellent qubits, gates, ...
- Conflicting requirements: good isolation from environment while maintaining good addressability

[M. Nielsen and I. Chuang, Quantum Computation and Quantum Information (Cambridge, 2000)]
Topics – superconducting qubits

- realization of superconducting quantum electronic circuits
  - harmonic oscillators (photons)
  - non-harmonic oscillators (qubits)
- controlled qubit/photon interactions
  - cavity quantum electrodynamics with circuits
- qubit read-out
- single qubit control
- decoherence
- two-qubit interactions
  - generation of entanglement (C-NOT gate)
  - realization of quantum algorithms (teleportation)
Classical and Quantum Electronic Circuit Elements

basic circuit elements:

charge on a capacitor:

\[ \frac{1}{\sqrt{2}} \left( \frac{}{} \right) \]

current or magnetic flux in an inductor:

quantum superposition states:

- charge q
- magnetic flux \( \phi \)
Constructing Linear Quantum Electronic Circuits

basic circuit elements:  harmonic LC oscillator:  energy:

- typical inductor: $L = 1 \text{nH}$
- a wire in vacuum has inductance $\sim 1 \text{nH/mm}$
- typical capacitor: $C = 1 \text{pF}$
- a capacitor with plate size $10 \mu\text{m} \times 10 \mu\text{m}$ and dielectric AlOx ($\varepsilon = 10$) of thickness $10 \text{nm}$ has a capacitance $C \sim 1 \text{pF}$

resonance frequency:

$$\omega = \frac{1}{\sqrt{LC}} \sim 5 \text{GHz}$$

Quantization of the electrical LC harmonic oscillator:

para llel LC oscillator circuit:

\[ V = \frac{Q}{C} = -L \frac{dI}{dt} \]

voltage across the oscillator:

\[ H = \frac{1}{2} CV^2 + \frac{1}{2} LI^2 = \frac{1}{2} \frac{Q^2}{C} + \frac{1}{2} \frac{\phi^2}{L} \]

total energy (Hamiltonian):

with the charge \( Q \) stored on the capacitor

\( Q = VC \)

a flux \( \phi \) stored in the inductor

\( \phi = LI \)

properties of Hamiltonian written in variables \( Q \) and \( \phi \):

\[ \frac{\partial H}{\partial Q} = \frac{Q}{C} = -L \frac{dI}{dt} = -\dot{\phi} \]

\[ \frac{\partial H}{\partial \phi} = \frac{\phi}{L} = I = \dot{Q} \]

\( Q \) and \( \phi \) are canonical variables

see e.g.: Goldstein, Classical Mechanics, Chapter 8, Hamilton Equations of Motion
Quantum version of Hamiltonian

\[ \hat{H} = \frac{\hat{\phi}^2}{2 \zeta} + \frac{\hat{\phi}^2}{2 \zeta} \]

with commutation relation

\[ [\hat{\phi}, \hat{\phi}] = i \hbar \]

compare with particle in a harmonic potential:

\[ \hat{H} = \frac{\hat{\mathbf{p}}^2}{2m} + \frac{1}{2} \kappa \omega^2 \hat{x}^2 \]

analogy with electrical oscillator:
- charge \( q \) corresponds to momentum \( \mathbf{p} \)
- flux \( \phi \) corresponds to position \( x \)

Hamiltonian in terms of raising and lowering operators:

\[ \hat{H} = \hbar \omega \left( a^+ a + \frac{1}{2} \right) \]

with oscillator resonance frequency:

\[ \omega = \frac{1}{\sqrt{\varepsilon C \varepsilon} \varepsilon} \]
Raising and lowering operators:

\[ a^+ (n) = \sqrt{n+1} \ (n+1) \ ; \ a^+ (n) = \sqrt{n} \ a (n-1) \]

\[ a^+ a \ (n) = n \ a (n) \]

number operator

in terms of \( \mathcal{Q} \) and \( \phi \):

\[ \hat{\mathcal{Q}} = \frac{1}{\sqrt{2 \hbar Z_c}} \ (Z_c \mathcal{Q} + i \ \mathcal{\phi}) \]

with \( Z_0 \) being the characteristic impedance of the oscillator

\[ Z_c = \sqrt{\frac{L_c}{C_c}} \]

charge \( \mathcal{Q} \) and flux \( \phi \) operators can be expressed in terms of raising and lowering operators:

\[ \hat{\mathcal{Q}} = \sqrt{\frac{\hbar}{2 Z_c}} \ (a^+ - a) \]

\[ \hat{\phi} = i \sqrt{\frac{\Phi_0}{2 Z_c}} \ (a^+ - a) \]

**Exercise:** Making use of the commutation relations for the charge and flux operators, show that the harmonic oscillator Hamiltonian in terms of the raising and lowering operators is identical to the one in terms of charge and flux operators.
Realization of H.O.: Lumped Element Resonator

Inductor \( L \), Capacitor \( C \)

A harmonic oscillator

Currents and magnetic fields

Charges and electric fields

\( \phi \)
Realization of H.O.: Transmission Line Resonator

distributed resonator:

- coplanar waveguide resonator
- close to resonance: equivalent to lumped element LC resonator

Transmission line resonator

optical microscope image of sample fabricated at FIRST (Nb on sapphire)

electric field across resonator in vacuum state ($n=0$):

$$E_{0,\text{rms}} \approx 0.2 \text{ V/m} \quad \text{for} \quad \omega_r/2\pi \approx 6 \text{ GHz}$$

$$H_r = \hbar \omega_r \left( a^\dagger a + \frac{1}{2} \right)$$
Why Superconductors?

normal metal    superconductor

- single non-degenerate macroscopic ground state
- elimination of low-energy excitations

Superconducting materials (for electronics):
- Niobium (Nb): \(2\Delta/h = 725\) GHz, \(T_c = 9.2\) K
- Aluminum (Al): \(2\Delta/h = 100\) GHz, \(T_c = 1.2\) K

How to make qubit?

Cooper pairs: bound electron pairs

Bosons (\(S=0, L=0\))

2 chunks of superconductors

macroscopic wave function

\[ \psi_i = \frac{1}{\sqrt{N}} e^{i\delta_i} \]

Cooper pair density \(n_i\) and global phase \(\delta_i\)

phase quantization: \(\delta = n \cdot 2\pi\)

flux quantization: \(\phi = n \cdot \phi_0\)

\(\phi_0\)… magnetic flux quantum (2.067 \(10^{-15}\) Wb)
How to prepare quantum states?

Question: What happens to the harmonic oscillator (in ground state), if we drive transitions at frequency $\omega$?

Transitions to higher levels will be driven equally, harmonic oscillator will be in a ‘coherent’ state, which is the most classical state

$\rightarrow$ no quantum features observables
Constructing Non-Linear Quantum Electronic Circuits

circuit elements:

anharmonic oscillator:

Josephson junction:

a non-dissipative nonlinear element (inductor)

\[ L_J(\phi) = \left( \frac{\partial I}{\partial \phi} \right)^{-1} \]

\[ = \frac{\phi_0}{2\pi I_c \cos(2\pi \phi/\phi_0)} \]

Linear vs. Nonlinear Superconducting Oscillators

LC resonator

Josephson junction resonator
Josephson junction = nonlinear inductor

energy

\begin{align*}
|0\rangle \\
|1\rangle \\
|2\rangle
\end{align*}

\begin{align*}
\Phi
\end{align*}

anharmonicity → effective two-level system

\begin{align*}
|e\rangle \\
|f\rangle \\
|g\rangle
\end{align*}
A Low-Loss Nonlinear Element

a (superconducting) Josephson junction

- superconductors: Nb, Al
- tunnel barrier: AlO$_x$

Josephson Tunnel Junction

the only non-linear LC resonator with no dissipation (BCS, $k_B T \ll \Delta$)

\[
Q = +N(2e) \quad -Q = -N(2e)
\]

\[1 \text{nm}\]

Josephson junction relations:
- critical current $I_0$
- junction capacitance $C_J$
- high internal resistance $R_J$ (insulator)

Josephson relation:
\[
I = I_0 \sin \delta
\]

\[
V = \Phi_0 \frac{\partial \delta}{\partial t}
\]

(reduced) flux quantum:
\[
\Phi_0 = \frac{\phi_0}{2\pi} = \frac{\hbar}{2e}
\]

phase difference:
\[
\delta = \delta_2 - \delta_1
\]

derivation of Josephson effect, see e.g.: chap. 21 in R. A. Feynman: Quantum mechanics, The Feynman Lectures on Physics. Vol. 3 (Addison-Wesley, 1965)
The Josephson junction as a non-linear inductor

induction law: \[ V = -L \frac{\partial I}{\partial t} \]

Josephson effect: ac-Josephson equation

\[ I = I_c \sin \delta \]
\[ \frac{\partial I}{\partial t} = I_c \cos \delta \frac{\partial \delta}{\partial t} \]

ac-Josephson equation

\[ V = \frac{\Phi_0}{2\pi} \frac{\partial \delta}{\partial t} = \frac{\Phi_0}{2\pi I_c} \frac{1}{\cos \delta} \frac{\partial I}{\partial t} \]

Josephson inductance

\[ L_J = \frac{\Phi_0}{2\pi I_c} \frac{1}{\cos \delta} \]

specific Josephson inductance

nonlinearity

A typical characteristic Josephson inductance for a tunnel junction with \( I_c = 100 \text{ nA} \) is \( L_J \sim 3 \text{ nH} \).

How to Operate Circuits Quantum Mechanically?

recipe:
- avoid dissipation
- work at low temperatures
- isolate quantum circuit from environment

Quantum Harmonic Oscillator at Finite Temperature

thermal occupation:

$$\langle n_{th} \rangle = \frac{1}{\exp(\hbar \nu / k_B T) - 1}$$

low temperature required:

$$\hbar \omega \gg k_B T$$

10 GHz ~ 500 mK

$$\langle n_{th} \rangle \sim 10^{-11}$$
Internal and External Dissipation in an LC Oscillator

- Internal losses: $R_{\text{int}}$
  - conductor, dielectric
- External losses: $R_{\text{ext}}$
  - radiation, coupling

Total losses:

$$\frac{1}{R} = \frac{1}{R_{\text{int}}} + \frac{1}{R_{\text{ext}}}$$

- Impedance: $Z = \sqrt{\frac{L}{C}}$
- Quality factor: $Q = \frac{R}{Z} = \omega_0 RC$
- Excited state decay rate: $\Gamma_1 = \frac{\omega_0}{Q} = \frac{1}{RC}$
Resonator Quality Factor and Photon Lifetime

resonance frequency:

\[ \nu_r = 6.04 \text{ GHz} \]

quality factor:

\[ Q = \frac{\nu_r}{\delta \nu_r} \approx 10^4 \]

photon decay rate:

\[ \frac{\kappa}{2\pi} = \frac{\nu_r}{Q} \approx 0.8 \text{ MHz} \]

photon lifetime:

\[ T_\kappa = \frac{1}{\kappa} \approx 200 \text{ ns} \]
Controlling Coupling to the E.M. Environment

Coupling to environment (bias wires):
- decoherence
  - from energy relaxation
    - (spontaneous emission)

Decoupling using non-resonant impedance transformers:

Using resonant impedance transformers

Control spontaneous emission by circuit design
How to Make Use of the Josephson Junction in Qubits?

different bias (control) circuits:

- **phase qubit**
- **charge qubit**
  \[ q_g = C_g V_g \]
- **flux qubit**

current bias

charge bias

flux bias