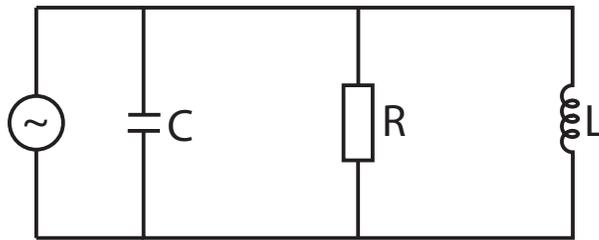


QSIT 2010 - Questions 5

March 30, 2012, HIT F13

1. Driven damped oscillator

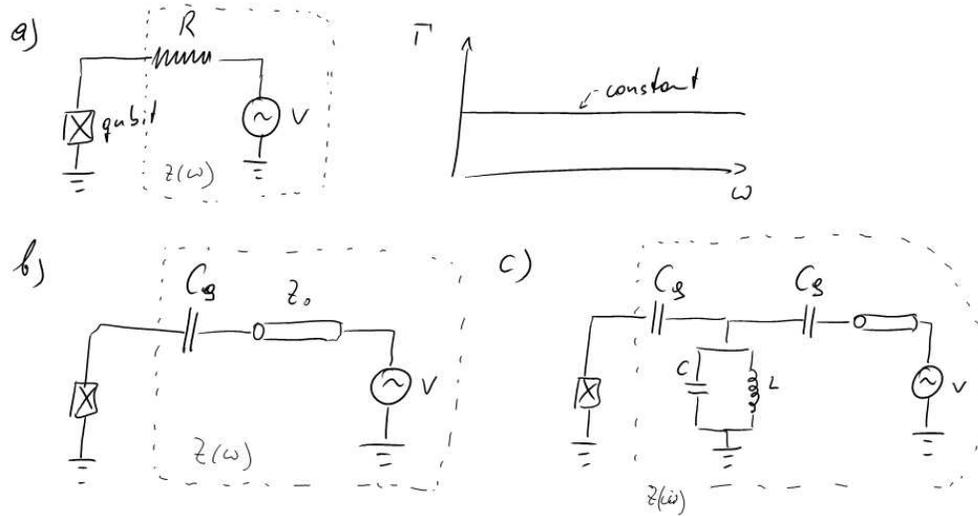
Consider an parallel LCR -resonator shown in the figure.



- Derive a differential equation describing the dynamics of this resonator.
- How does the energy stored in the oscillator depend on the drive frequency?
- Calculate the energy decay rate.

2. Energy relaxation of a qubit

In analogy to the harmonic oscillator, the energy decay time of a qubit is given by $T_1 = RC$, where C is the intrinsic capacitance of the qubit. R denotes the effective resistance $R = 1/\text{Re}[Y(\omega)]$ obtained from the impedance of the environment $Z(\omega) = 1/Y(\omega)$ as seen from the position of the qubit.



If the impedance of the environment is purely resistive, e.g. $Z(\omega) = 50 \Omega$, the decay rate $\Gamma = 1/T_1$ is frequency independent (see Figure a).

- Derive the impedance of a Cooper-pair box qubit that is capacitively coupled to a transmission line ($Z_0 = 50 \Omega$) via a gate capacitance C_g (Figure b). Sketch the decay rate Γ as a function of frequency.
- What is the spectral shape of Γ for a coupling to an LC oscillator (Figure c)?

3. Superconducting loop

SQUID stands for Superconducting Quantum Interference Device and consists of a superconducting loop intersected by two Josephson junctions as shown in the figure below. In general the junction is described by its phase ϕ which depends on the supercurrent flowing through it according to

$$I = I_c \sin \phi, \quad (1)$$

where I_c is the critical current of the junction. In addition to Eq. (1) we can relate magnetic flux Φ through the loop and all currents by Kirchhoff rules:

$$I_2 + I_1 = I, \quad (2)$$

$$\Phi + \frac{\Phi_0}{2\pi}(\phi_1 - \phi_2) = 0 \quad (3)$$

- Show that the total current I is related to the average phase $\phi_p = (\phi_1 + \phi_2)/2$ by the current-phase relationship similar to (1) with the

critical current modulated by the magnetic flux through the loop $I_c(\Phi)$. Assume both junctions to be identical.

