2.7. Circuit Quantum Electrodynamics

Donnerstag, 22. März 2012
07:59

vacuum field $\mathcal{E}_0$:

EM - energy in vacuum state: $\frac{1}{2}\hbar\omega_c$

($\frac{1}{2}$ magnetic, $\frac{1}{2}$ electric)

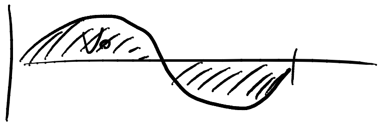
from E-dyn: $W = \frac{\varepsilon_0}{2}\int_V |E|^2 dV \stackrel{\text{Q.M.}}{\hat{=}} \frac{\varepsilon_0}{2}\int \langle 0||\hat{E}|^2|0\rangle \, dV$

$$\hat{E}(r) = \mathcal{E}_0 f(r) \cdot (\hat{a}^\dagger + \hat{a})$$

$$|\hat{E}|^2 = \mathcal{E}_0^2 f(r)^2 \left(\hat{a}^\dagger\hat{a}^\dagger + \hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a} + \hat{a}\hat{a}\right)$$

$$\langle 0||E|^2|0\rangle = \mathcal{E}_0^2 f(r)^2 \quad \Rightarrow \quad \frac{1}{4}\cdot\hbar\omega_c = \frac{\varepsilon_0}{2}\cdot\mathcal{E}_0^2\cdot V_0$$

mode volume: $V_0 = \int f(r)^2 d^3r$



$$\boxed{\mathcal{E}_0 = \sqrt{\frac{\hbar\omega_c}{2\varepsilon_0 V_0}}}$$

$\Rightarrow$ small mode volume $\Rightarrow$ high electric fields

$\Rightarrow$ large coupling

1D cavity: $\mathcal{E}_0 \sim 0.2\,V/m$ ($V_0 \sim 5\cdot 10^{-3}\,mm^3$, $\nu_c \sim 6\,GHz$)

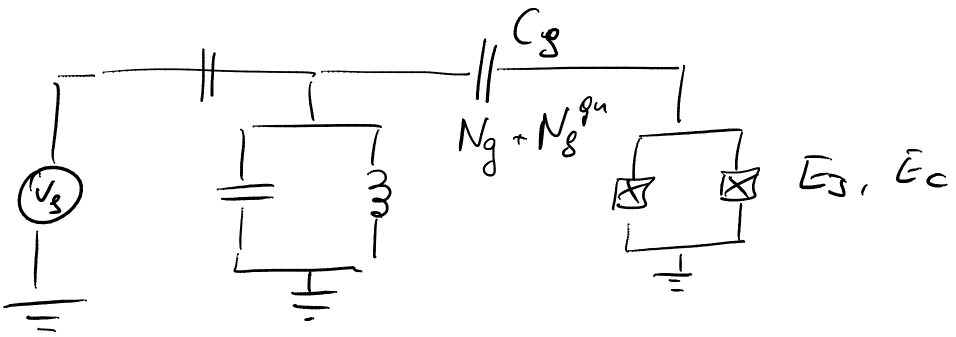
3D cavity: $E_0 \sim \sqrt{\frac{10^{-34}\cdot 2\pi\, 50\cdot 10^{9}}{2\cdot 8\cdot 10^{-12}\cdot 700\cdot 10^{-9}}} \sim 0.002\,V/m$

($V_0 \sim 700\,mm^3$)

($\nu_c \sim 50\,GHz$)

2.8. Jaynes Cummings Hamiltonian -Circuit QED

Donnerstag, 27. Oktober 2011
17:33



N_g .. polarization charge on C_g

$$N_g = \frac{C_g V_g}{2e}$$

$$\hat{H} = \underbrace{\hbar\omega_1\left(a^\dagger a + \frac{1}{2}\right)}_{H.O.} + \underbrace{\frac{E_C}{2}\left(1 - 2\left(N_g + N_g^{qm}\right)\right)\hat{\sigma}_z - \frac{E_J}{2}\hat{\sigma}_x}_{H\ C.P.B.}$$

charge degeneracy: $N_g = \frac{1}{2}$:

$$H_{CPB} = E_C N_g^{qm}\sigma_z - \frac{E_J}{2}\sigma_x$$

with $N_g^{qm} = \frac{\hat{Q}}{2e} = \frac{C_g}{2e}\hat{V} = \frac{C_g}{2e}\sqrt{\frac{\hbar\omega}{2C}}\left(\hat{a}^\dagger + \hat{a}\right)$

and change of basis: $\hat{\sigma}_z \rightarrow \hat{\sigma}_x$ and $\hat{\sigma}_x \rightarrow -\hat{\sigma}_z$

$$H_{CPB} = \frac{E_C C_g}{2e}\sqrt{\frac{\hbar\omega}{2C}}\left(\hat{a}^\dagger + \hat{a}\right)\hat{\sigma}_x + \frac{E_J}{2}\hat{\sigma}_z$$

qubit raising and lowering operators: $\sigma_x = \hat{\sigma}^+ + \hat{\sigma}^-$

interaction term

$$\frac{E_c C_g}{2e}\sqrt{\frac{\hbar\omega}{2C}}\left(\cancel{\hat{a}^+\hat{\sigma}^+} + \hat{a}^+\hat{\sigma}^- + \hat{a}\hat{\sigma}^+ + \cancel{\hat{a}\hat{\sigma}^-}\right)$$

rotating wave approximation (RWA)
not energy conserving

$$\Rightarrow \hat{H} = \hbar\omega_r\left(a^+a + \frac{1}{2}\right) + \hbar g\left(\hat{a}^+\hat{\sigma}^- + \hat{a}\hat{\sigma}^+\right) + \frac{E_J}{2}\hat{\sigma}_z$$

with coupling constant $\hbar g = \frac{C_g}{C_\Sigma} e\sqrt{\frac{\hbar\omega}{2C}}$

$\frac{2g}{2\pi}$ ... vacum Rabi frequency

2.9. Qubit Drive

Montag, 26. März 2012
15:01

Qubit drive:

apply time-dependent signal to CPB by changing N_g
(classical coherent signal)

$$H = -\frac{E_C}{2}\left(1 - 2\left[N_g + \eta \cos\omega t\right]\right)\sigma_z - \frac{E_J}{2}\sigma_x$$

$$= -\frac{E_C}{2}(1-2N_g)\,\sigma_z + \underbrace{E_C\eta}_{\varepsilon}\cos\omega t\,\sigma_z - \frac{E_J}{2}\sigma_x$$

for $N_g = \frac{1}{2}$ + swap of basis:

$$\boxed{H = \frac{E_J}{2}\sigma_z + \varepsilon\cos\omega t\,\sigma_x}$$