Experimental implementation of Grover's algorithm with transmon qubit architecture

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What is Grover's algorithm?

• Quantum search algorithm

• Task: In a search space of dimension N, find those 0<M<N elements displaying some given characteristics (being in some given states).

<table>
<thead>
<tr>
<th>Classical search (random guess)</th>
<th>Grover’s algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Guess randomly the solution</td>
<td>• Apply an ORACLE, which marks the solution</td>
</tr>
<tr>
<td>• Control whether the guess is actually a solution</td>
<td>• Decode the marked solution, in order to recognize it</td>
</tr>
<tr>
<td>O(N) steps</td>
<td>O((\sqrt{N})) x n steps</td>
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<tr>
<td>O(N) bits needed</td>
<td>O(log(N)) qubits needed</td>
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</table>
The oracle

- The oracle MARKS the correct solution
  
  \[ f(x) = \begin{cases} 
  0 & \text{x is not solution} \\
  1 & \text{x is solution}
  \end{cases} \]

- Solution is **more** recognizable

\[ \text{Dilution operator (interpreter)} \]
Grover's algorithm

Procedure

• Preparation of the state

\[ |\psi\rangle = \frac{1}{\sqrt{N}} \sum_x |x\rangle \]

• Oracle application

\[ |x\rangle \xrightarrow{O} (-1)^{f(x)} |x\rangle \]

• Dilution of the solution

\[ 2|\psi\rangle\langle\psi| - I \]

• Readout
Geometric visualization

- Preparation of the state

\[ |\psi\rangle = \frac{1}{\sqrt{N}} \sum_{x} |x\rangle \]

\[ |\bar{t}\rangle = \frac{1}{\sqrt{N - M}} \sum_{x}^{' |x\rangle} \]

\[ |t\rangle = \frac{1}{\sqrt{M}} \sum_{x}^{''} |x\rangle G|\psi\rangle \]

\[ |\psi\rangle = \sqrt{\frac{N - M}{N}} |\alpha\rangle + \sqrt{\frac{M}{N}} |\beta\rangle \]
Geometric visualization

\[ |\psi\rangle = \cos(\theta/2)|\bar{t}\rangle + \sin(\theta/2)|t\rangle \]

\[ O|\psi\rangle = \cos(\theta/2)|\bar{t}\rangle - \sin(\theta/2)|t\rangle \]

\[ 2|\psi\rangle\langle\psi| - I \]

\[ G|\psi\rangle = \cos(\frac{3\theta}{2})|\bar{t}\rangle + \sin(\frac{3\theta}{2})|t\rangle \]
Grover's algorithm
Performance

- Every application of the algorithm is a rotation of $\theta$

$$G^k|\psi\rangle = \cos\left(\frac{(2k+1)\theta}{2}\right)|\bar{t}\rangle + \sin\left(\frac{(2k+1)\theta}{2}\right)|t\rangle$$

- The ideal number of rotations is:

$$R = C I \left[\frac{\arccos \sqrt{M/N}}{\theta}\right]$$
Grover's algorithm
2 qubits

N=4
Oracle marks one state M=1

\[ |\psi\rangle = \frac{1}{2} (|00\rangle + |01\rangle + |10\rangle + |11\rangle) \]

\[ = \frac{1}{2} |t\rangle + \frac{\sqrt{3}}{2} \frac{1}{\sqrt{3}} (|t_1\rangle + |t_2\rangle + |t_3\rangle) \]

\[ = \frac{1}{2} |t\rangle + \frac{\sqrt{3}}{2} |\bar{t}\rangle \]

After a single run and a projection measurement will get target state with probability 1!
Grover's algorithm

Circuit

\[ |\psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \]
Grover's algorithm

Oracle

Will get 2 cases:

\[ e^{\pm i\pi} = \pm i \]
\[ e^{\pm i0} = 1 \]
Grover's algorithm

Decoding

\[
\frac{1}{\sqrt{2}}\left(\begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \end{array}\right)
\]

for \( t = 2 \)

1. \( \frac{1}{2}\left(\begin{array}{c} -1 \\ 1 \\ 1 \\ 1 \end{array}\right) \) iSWAP
2. \( \frac{1}{2}\left(\begin{array}{c} -1 \\ -i \\ -i \\ 1 \end{array}\right) = -\frac{1}{\sqrt{2}} (|0\rangle + i|1\rangle) \otimes \frac{1}{\sqrt{2}} (|0\rangle + i|1\rangle) \)

\[
= |\psi\rangle \otimes |\psi\rangle
\]

3. \( \frac{1}{2}\left(\begin{array}{c} 1 \\ -1 \\ 1 \\ 1 \end{array}\right) \) iSWAP
4. \( \frac{1}{2}\left(\begin{array}{c} 1 \\ 1 \\ i \\ -i \end{array}\right) = |\psi\rangle \otimes |\psi\rangle
\]

5. \( \frac{1}{2}\left(\begin{array}{c} 1 \\ 1 \\ 1 \\ -1 \end{array}\right) \) iSWAP
6. \( \frac{1}{2}\left(\begin{array}{c} 1 \\ -i \\ -i \\ 1 \end{array}\right) = |\psi\rangle \otimes |\psi\rangle
\]
Experimental setup

$\omega_q =$ qubit frequency
$\omega_r =$ resonator frequency
$\omega =$ rotating frame frequency
$\Omega =$ drive pulse amplitude
Single qubit manipulation

- Qubit frequency control via flux bias
- Rotations around z axis: detuning $\Delta$
- Rotations around x and y axes: resonant pulses $\Omega$

$$H_{rot} = \frac{\omega_a - \omega}{2} \sigma_z + \frac{\Omega}{2} (\cos \phi \sigma_x + \sin \phi \sigma_y) \equiv \frac{\delta}{2} \sigma_z + \frac{\Omega_x}{2} \sigma_x + \frac{\Omega_y}{2} \sigma_y.$$
Experimental setup

ω_{q, I} = qubit frequency
ω_{q, II} = resonator frequency
g = coupling strength
Qubit capacitive coupling

In the rotating frame

\[ \omega = \omega_{q,II} \]

the coupling Hamiltonian is:

\[ H_{\text{tot}} = h(\omega_{q,I}\sigma_z^I + \omega_{q,II}\sigma_z^{II} + H_{\text{int}}) \]

\[ H_{\text{int}} = g(|10\rangle\langle01| + |01\rangle\langle10|) \]
iSWAP gate

- Controlled interaction between $\langle 10 |$ and $\langle 01 |$

- By letting the two states interact for $t = \pi/g$ we obtain an iSWAP gate!

$$U_{\text{int}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(gt/2) & -i\sin(gt/2) & 0 \\ 0 & -i\sin(gt/2) & \cos(gt/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
Pulse sequence
Linear transmission line and single-shot measurement

Quality factor of empty linear transmission line

\[ Q = \frac{\nu_r}{\Delta \nu_r} \]

With resonant frequency \( \nu_r \).

Presence of a transmon in \( |g\rangle \) state shifts the resonant frequency of the transmission line \( \omega_0 = 2\pi \nu_r \rightarrow \omega_0 - \chi \)

If microwave at \( \omega_0 - \chi \) full transmission for \( |g\rangle \) state, partial for \( |e\rangle \) state.

Current corresponding to transmitted EM

\[ \alpha \]

10^6 averages

\( g \)

\( e \)

\( \omega_0 + \chi \)

\( \omega_0 \)

\( \omega_0 - \chi \)

\( \omega_m \)

\( 1/\kappa \sim 0.1 \mu s \)

\( T_1 \sim 1 \mu s \)

Excited state response

Ground state response

Time [\mu s]
Linear transmission line and single-shot measurement

Single shot
- If there was no noise, would get either blue or red curve
- Real curve so noisy that cannot tell whether $|g\rangle$ or $|e\rangle$

Cannot do single-shot readout

We need an amplifier which increases the area between $|g\rangle$ and $|e\rangle$ curves, but does not amplify the noise
Josephson Bifurcation Amplifier (JBA)

Nonlinear transmission line due to Josephson junction
Resonant frequency $\omega_0$

At $P_{\text{in}} = P_C$ max. slope diverges

Bifurcation: at the correct $(P_{\text{in}}, \omega_d)$ two stable solutions, can map the collapsed state of the qubit to them
Josephson Bifurcation Amplifier (JBA)

Switching probability $p$: probability that the JBA changes to the higher-amplitude solution

- Excite $|1\rangle \rightarrow |2\rangle$
- Choose power corresponding to the biggest difference of switching probabilities

Errors
- Nonzero probability of incorrect mapping
- Crosstalk
Errors

- Nonzero probability of incorrect mapping
- Crosstalk
Conclusions

• Gate operations of Grover algorithm successfully implemented with capacitively coupled transmon qubits

• Arrive at the target state with probability 0.62 – 0.77 (tomography)

• Single-shot readout with JBA (no quantum speed-up without it)

• Measure the target state in single shot with prob 0.52 – 0.67 (higher than 0.25 classically)
Sources

• Dewes, A; Lauro, R; Ong, FR; et al., "Demonstrating quantum speed-up in a superconducting two-qubit processor”, arXiv:1109.6735 (2011)

• Bialczak, RC; Ansmann, M; Hofheinz, M; et al., “Quantum process tomography of a universal entangling gate implemented with Josephson phase qubits”, Nature Physics 6, 409 (2007)
Thank you for your attention

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