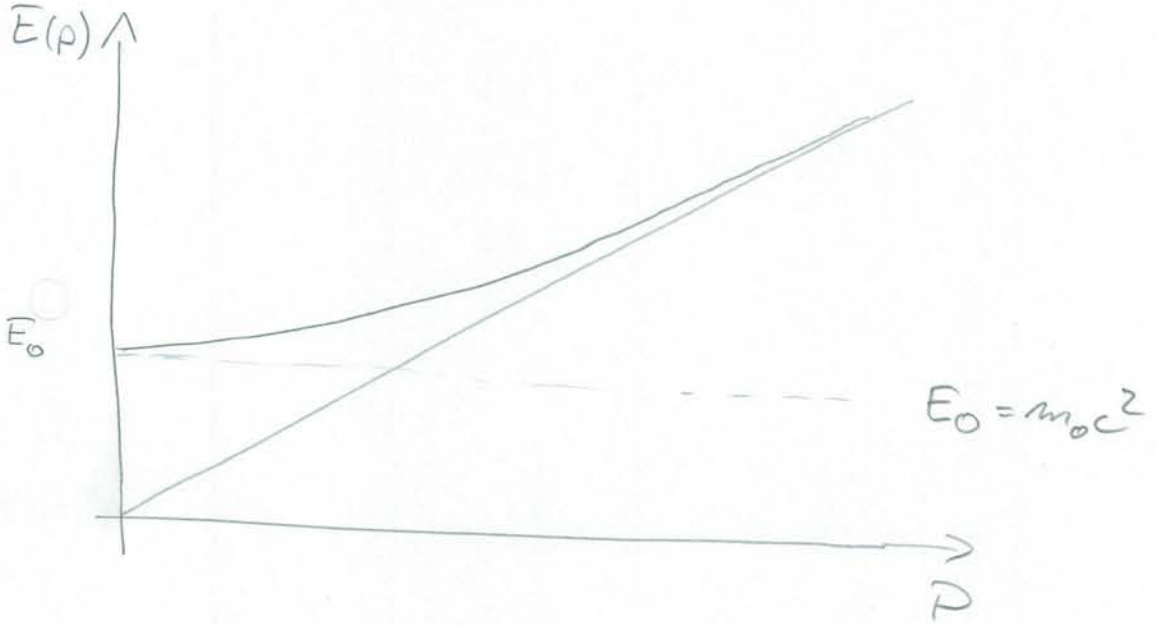
Dispersion von Materiewellen

$$\psi(x,t) = \psi_0 e^{-i(kx-\omega t)}$$

$$= \psi_0 e^{-i/\hbar (px - Et)}$$



$$E(p) = \sqrt{m_0^2 c^4 + p^2 c^2}$$

- Grenzfall $v \ll c$: (nicht relativistischer Grenzfall)

$$E = m_0 c^2 \sqrt{1 + \frac{p^2 c^2}{m_0^2 c^4}}$$

$$\approx m_0 c^2 \left(1 + \frac{p^2 c^2}{2 m_0^2 c^4}\right) = m_0 c^2 + \frac{p^2}{2 m_0}$$

$m_0 c^2 = E_0$; $\frac{p^2}{2 m_0} = E - E_0 = E_{kin}$

$$E_{kin} = \frac{p^2}{2 m_0} = \frac{\hbar^2 k^2}{2 m_0} \propto k^2 \Rightarrow \text{Dispersion}$$

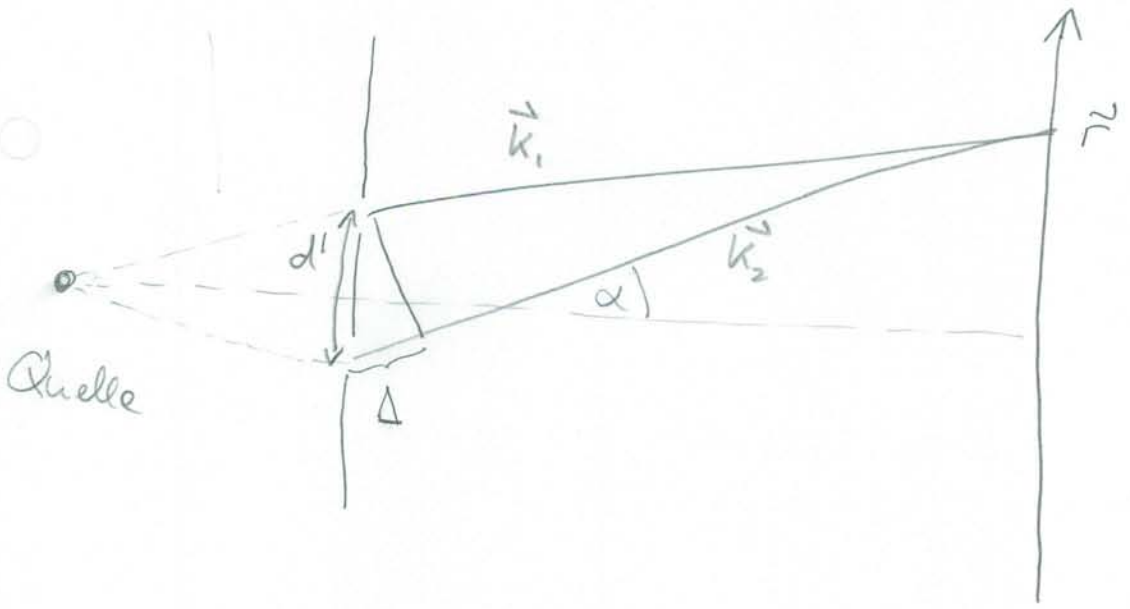
$$\frac{\partial E}{\partial p} \neq \text{const}$$

$$\Rightarrow \lambda = \frac{h}{p} = \frac{h}{\sqrt{2 m_0 E_{kin}}}$$

Interferenz von Materiewellen

$$\psi = \underbrace{\psi_0}_{\text{Amplitude}} e^{-i(\underbrace{\vec{k}\vec{r} - \omega t}_{\text{Phase}})}$$

Doppelspalt:



$\Delta = \sin\alpha \, d'$ Gangunterschied

$\delta = 2\pi \frac{\Delta}{\lambda}$ Phasenunterschied

$\delta = n\,2\pi$ $\Rightarrow$ konstruktive Interferenz

$\delta = (2n+1)\,\pi$ $\Rightarrow$ destruktive Interferenz

Interferenzmuster sind analog zu denen der optischen Interferenz am Doppelspalt

Berechnung des Interferenzmusters

$$|\Psi|^2 = |\Psi_1 + \Psi_2|^2 \quad \text{mit} \quad \Psi_1 = \Psi_0 e^{-i\vec{k}_1\vec{r}}$$
$$\Psi_2 = \Psi_0 e^{-i\vec{k}_2\vec{r}}$$

$$= (\Psi_1 + \Psi_2)(\Psi_1 + \Psi_2)^*$$

$$= \Psi_1\Psi_1^* + \Psi_2\Psi_2^* + \overbrace{\Psi_1\Psi_2^* + \Psi_1^*\Psi_2}^{\text{Interferenzterm}}$$

$$= |\Psi_0|^2 + |\Psi_0|^2 + 2|\Psi_0||\Psi_0|\cos\delta$$

mit Phasendifferenz δ

$$|\Psi|^2 = \begin{cases} 4|\Psi_0|^2 & \text{für } \delta = m\,2\pi \\ 0 & \text{für } \delta = (2m+1)\,\pi \end{cases}$$