

# Observation of whispering gallery resonances in annular Josephson junctions

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## Abstract

Experiments indicating the excitation of whispering gallery type electromagnetic modes by a vortex moving in an annular Josephson junction are reported. At relativistic velocities the Josephson vortex interacts with the modes of the superconducting stripline resonator giving rise to novel resonances on the current-voltage characteristic of the junction. The coupling between the vortex and the linear modes of the junction is explained using a Cherenkov-radiation like mechanism. We analytically calculate the spectrum of the linear modes taking into account the electromagnetic environment of the junction. The experimental data are in good quantitative agreement with analysis and numerical calculations based on the two-dimensional sine-Gordon model.

*Keywords:* long Josephson junctions; Josephson vortices; whispering gallery modes; Josephson plasma waves

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Whispering gallery modes are universal linear excitations of circular and annular resonators. They have first been observed in form of a sound wave traveling along the outer wall of a walkway in the circular dome of St. Paul's Cathedral in London and were investigated by Lord Rayleigh [1] and others. Referring to the acoustic phenomenon the name "whispering gallery modes" was also coined for electromagnetic eigenmodes of circular resonators which can be described in terms of high order Bessel functions. We report on the observation of such electromagnetic modes in annular

Josephson junctions, where they are excited by the relativistic motion of Josephson vortices.

An annular long Josephson junction is an intriguing nonlinear wave propagation medium for the experimental study of the interaction between linear waves and solitons [2]. Resonant interaction between topological solitons (Josephson vortices) and the linear whispering gallery modes of the annular junction was recently predicted theoretically [3]. In our experiment, we make use of the same Josephson vortex for both exciting and detecting the whispering gallery mode. This interaction is manifested by the presence of a novel fine structure on the current-voltage characteristic (IVC) of the junction, which depends sensitively on the electri-

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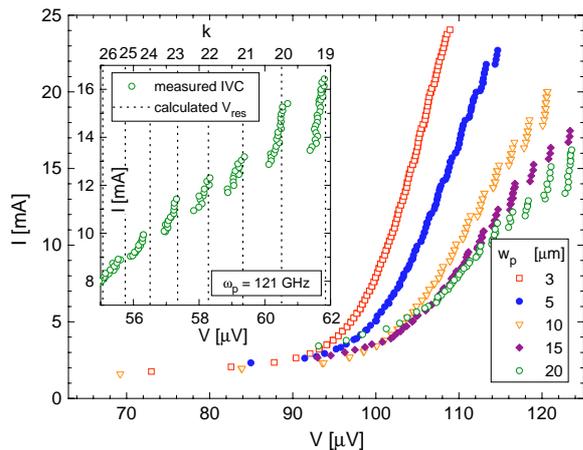


Fig. 1. Current-voltage characteristics of vortex-antivortex state in junctions of different  $w_p$ . The calculated resonance voltages and the IVC for  $w_p = 20 \mu\text{m}$  are compared in the inset. The top axes indicates the angular wave number of the excited mode.

cal parameters of the Josephson junction and its environment.

We have designed a set of annular junctions [4] with mean radius  $55 \mu\text{m}$ , junction width  $10 \mu\text{m}$  and varying width of the passive region around the junction  $w_p = 3 \dots 20 \mu\text{m}$ . The passive region (often also called idle or window) is formed by the overlap of the junction electrodes, separated by an insulator ( $\text{SiO}_x$ ), surrounding the active tunnel junction area. In these samples we have observed a clear fine structure on the vortex-antivortex resonance at 4.2 K, see Fig. 1. Evidently, the fine structure step spacing does strongly depend on  $w_p$ . This observation can be explained considering the linear mode spectrum of the junction coupled to the idle region and the generation of Cherenkov radiation by the Josephson vortex into these modes, as predicted in [3].

The propagation of both Josephson vortices and linear modes in an annular Josephson junction surrounded by an idle region can be described by the perturbed sine-Gordon equation

$$\ddot{\varphi} - \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \varphi}{\partial r} - \frac{1}{r^2} \frac{\partial^2 \varphi}{\partial \theta^2} + \sin \varphi = \gamma - \alpha \dot{\varphi}, \quad (1)$$

coupled to a linear wave equation

$$\frac{1}{v^2} \ddot{\psi} - \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \psi}{\partial r} - \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} = 0 \quad (2)$$

through the boundary condition at the interface

$$\frac{\partial \varphi}{\partial r} = \lambda \frac{\partial \psi}{\partial r}. \quad (3)$$

Here Eqs. (1,2) are expressed in the usual normalization [2].  $v$  is the ratio of wave velocities and  $\lambda$  the ratio of inductances between the passive and the active regions of the junction. Linearizing Eq. (1) and using the boundary conditions the whispering gallery mode spectrum  $\omega(k)$  of the Josephson junction with idle can be calculated. The resonance voltages of the vortex motion and the linear waves can then be found [3]

$$V_{res} = \Phi_0 \frac{\omega_p}{2\pi} \frac{\omega_k}{k}. \quad (4)$$

The calculated voltages are compared to the experimental data for the junction with  $w_p = 20 \mu\text{m}$  in the inset of Fig. 1. The plasma frequency  $\omega_p = 121 \text{ GHz}$  determined by the fit is in good agreement with that evaluated from the critical current density  $j_c = 1 \text{ kA/cm}^2$ . At the highest step, the whispering gallery mode of the lowest angular wavenumber  $k = 19$  is excited. The wavenumber  $k$  of the excited mode increases incrementally from step to step while reducing the bias current.

We have shown that the linear mode spectrum of an annular Josephson junction resonator can be very sensitively probed by studying its interaction with Josephson vortices moving at relativistic velocities. The presented measurement technique can be complemented by spatially resolved imaging methods and direct radiation measurements to gain further experimental insight into the properties of such nonlinear sub-mm annular resonators.

## References

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