Quantum Dissociation of a Vortex-Antivortex Pair in a Long Josephson Junction

M. V. Fistul,[1] A. Wallraff,[1,2] Y. Koval,[1] A. Lukashenko,[1] B. A. Malomed,[3] and A. V. Ustinov[1]

[1]*Physikalisches Institut III, Universität Erlangen-Nürnberg, D-91058 Erlangen, Germany*

[2]*Department of Applied Physics, Yale University, New Haven, Connecticut 06520, USA*

[3]*Department of Interdisciplinary Science, Faculty of Engineering, Tel Aviv University, Tel-Aviv 69978, Israel*

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The thermal and the quantum dissociation of a single vortex-antivortex (VAV) pair in an annular Josephson junction is experimentally observed and theoretically analyzed. In our experiments, the VAV pair is confined in a pinning potential controlled by external magnetic field and bias current. The dissociation of the pinned VAV pair manifests itself in a switching of the Josephson junction from the superconducting to the resistive state. The observed temperature and field dependence of the switching current distribution is in agreement with the analysis. The crossover from the thermal to the macroscopic quantum tunneling mechanism of dissociation occurs at a temperature of about 100 mK. We also predict the specific magnetic field dependence of the oscillatory energy levels of the pinned VAV state.

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Great attention has been attracted to experimental and theoretical studies of *macroscopic quantum phenomena* in diverse Josephson coupled systems [1–4]. Most of these systems, e.g., dc biased single Josephson junctions (JJs), various SQUIDs, and small Josephson junction arrays, contain a few *lumped* Josephson junctions. Hence, they can be described by a few degrees of freedom (Josephson phases). At low temperatures, quantum-mechanical effects such as macroscopic quantum tunneling, energy level quantization [2,3], and coherent oscillations [4] of the Josephson phase have been observed.

As we turn to *spatially extended* Josephson systems, e.g., quasi-one-dimensional long Josephson junctions, parallel arrays, and Josephson junction ladders, which present a particular case of interacting many particle systems, the analysis and observation of macroscopic quantum dynamics become more complex. These systems support diverse nonlinear excitations, such as Josephson vortices (magnetic fluxons) and vortex-antivortex (VAV) pairs, which interact with inhomogeneities and linear (plasma) modes [5]. The classical dynamics of such excitations is well established and, in particular, the thermal fluctuation-induced escape of the Josephson phase from a metastable state has been studied [6,7]. However, while various macroscopic quantum-mechanical effects have been predicted [8–13], only a few of them were observed in experiments. In particular, studying the Josephson phase escape from the metastable state, the macroscopic quantum tunneling of many vortices has been observed in [14], and recently tunneling of a single vortex and its energy level quantization have been measured [15].

As the next step in the study of macroscopic quantum effects, in this Letter we report the thermal and the quantum dissociation of a *single vortex-antivortex pair*. States containing many VAV pairs are relevant to thin superconducting films or large two-dimensional Josephson arrays close to the Kosterlitz-Thouless transition [1]. A single VAV pair naturally appears in a long *annular* JJ placed in an external magnetic field H parallel to the junction plane [16,17] (Figs. 1(a) and 1(b)). For experiments, we fabricated a junction of diameter $d = 100$ μm and width $w = 0.5$ μm which was etched from a sputtered $Nb/AlO_x/Nb$ thin film trilayer and patterned using electron-beam lithography [18]. Its critical current density is 220 A/cm^2, the Josephson length is $\lambda_J \approx 30$ μm, and the normalized junction length is $L \equiv \pi d/\lambda_J \approx 10.5$. The measured magnetic field dependence of the switching current is shown in Fig. 1(c). In the field range $|H| < 1.5$ Oe (main central lobe), the switching of the JJ from the superconducting state to the resistive one occurs through the *dissociation* of a single field-induced VAV pair confined in the potential well created by an externally applied magnetic field and a dc bias current. This process is confirmed by direct numerical simulations of the full sine-Gordon equation for an annular JJ [16,17] of length L. The numerically found magnetic field dependence of the switching current is in excellent agreement with the measurement [see solid line of Fig. 1(c)]. Simulations of the magnetic field distribution in the junction clearly show the nucleation and subsequent dissociation of the VAV pair [see Fig. 1(d)]. Fluctuations, thermal and quantum, induce internal oscillations of the confined pair. At high temperature the dissociation then takes place in a form of thermal activation over the barrier. At low temperature, *macroscopic quantum tunneling* through the barrier occurs. At fields $|H| > 1.5$ Oe, the system becomes bistable as a well-separated VAV pair penetrates in the junction. This state is perfectly reproduced by our numerical calculations [see the first side lobes in Fig. 1(c)] and its quantum dynamics will be discussed elsewhere.

Next, we analyze the penetration and following dissociation of a VAV pair in the presence of a small magnetic field H and a large dc bias, namely, $\delta = (I_{c0} - I)/I_{c0} \ll 1$,

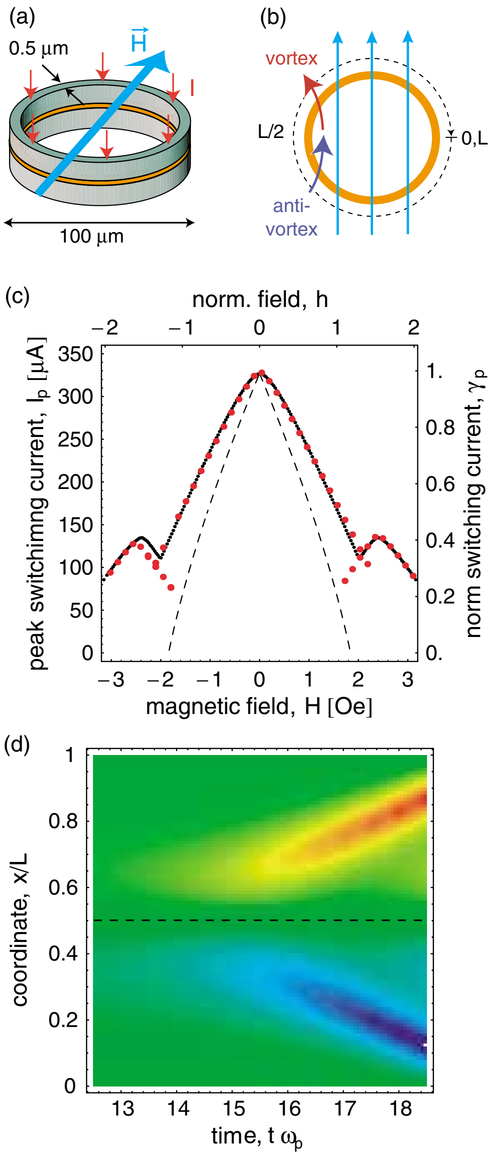

FIG. 1 (color online). (a) Schematic view of a long annular Josephson junction *without trapped vortices* in an in-plane external magnetic field H with uniform bias current I. (b) Generation of a confined vortex-antivortex pair with the center coordinate at $x = L/2$. (c) Magnetic field dependence of switching currents: experimental data at $T = 100$ mK (circles), numerical calculation for $L = 10.5$ (solid line), and theoretical prediction of Eq. (10) (dashed line). (d) Numerically simulated evolution of the magnetic field distribution, as the Josephson junction switches into the resistive state. The color (gray) scale corresponds to the magnitude of magnetic field in the junction. The emerging vortex and antivortex move in opposite directions.

where I_{c0} is the critical current of a long JJ for $H = 0$. In this case the Josephson phase is written as $\varphi(x, t) = \frac{\pi}{2} + \xi(x, t)$, where a small variable part ($|\xi(x, t)| \ll 1$) satisfies the following equation:

$$\xi_{tt} - \xi_{xx} - \frac{\xi^2}{2} = -\delta - h\cos\left(\frac{2\pi x}{L}\right), \qquad (1)$$

where $h \propto H$ is the normalized external magnetic field. Here, the coordinate x and time t are normalized to the Josephson penetration length λ_J and the inverse plasma frequency ω_p^{-1}, respectively. A particular solution of this equation satisfying the boundary conditions $\xi_x = 0$ at $x = \pm\infty$ is written in the form

$$\xi(|x - x_1|, A) = \sqrt{2\delta}\left[\frac{3}{\cosh^2(\frac{|x-x_1|+A}{2}(2\delta)^{1/4})} - 1\right]. \qquad (2)$$

Here, $x_1(t)$ is the center coordinate of a confined pair, and the parameter $A(t)$, which will be allowed to vary in time, determines the distance between vortex and antivortex. The magnetic field and dc bias create a pinning potential for such a state. Assuming that the JJ length L is much larger than the size of the pair l_p (which is $\simeq \delta^{-1/4}$), we substitute (2) in the Hamiltonian of the underlying sine-Gordon model and minimize it with respect to x_1, which readily amounts to setting $x_1 = L/2$. Then, we find an effective energy of the JJ,

$$E(A) = \frac{m_{\text{eff}}(A)}{2}\dot{A}^2 + U_{\text{pot}}(A),$$

as a function of A ($\dot{A}$ stands for the time derivative), where

$$U_{\text{pot}}(A) = m_{\text{eff}}(A) - 12(2\delta)^{1/4}h\tanh\left(\frac{A(2\delta)^{1/4}}{2}\right), \qquad (3)$$

with the effective mass of the VAV pair,

$$m_{\text{eff}}(A) = 18(2\delta)^{3/2}\int_{-\infty}^{A} dx\frac{\sinh^2(\frac{x(2\delta)^{1/4}}{2})}{\cosh^6(\frac{x(2\delta)^{1/4}}{2})}. \qquad (4)$$

In the absence of fluctuations, the critical current, $\delta_c(h) = 2h/3$, is found by minimization of the energy $E(A)$ in A. The corresponding critical value A_0 of A is determined by the condition $\sinh[(2\delta)^{1/4}A_0/2] = 1$. Note that the critical current decreases linearly with the magnetic field, in contrast to the case of *linear* long JJs, where similar consideration yields $\delta_c^{\text{lin}}(h) \propto h^{4/3}$. Both results are valid for the ideal uniform bias-current distribution and $L/2\pi \gg 1$.

In the presence of thermal or quantum fluctuations, the dissociation of the pinned VAV pair occurs at a random value of dc bias, i.e., at $\delta \geq \delta_c(h)$. Assuming weak fluctuations, $\delta - \delta_c(h) \ll \delta_c(h)$, we expand the energy of the state around $A = A_0$ ($\delta A = A - A_0$) as

$$E(A) = \frac{\chi h^{5/4}(\delta\dot{A})^2}{2} + \frac{3^{3/2}\sqrt{h}}{2}(\delta - \delta_c(h))(\delta A) - \frac{h^2}{6}(\delta A)^3,$$

where χ is a numerical coefficient of order one. Thus, the problem of the fluctuation-induced dissociation of a confined VAV pair is mapped into the well-known problem of particle escape from a cubic potential. The probability of the dissociation depends on the height of the effective potential barrier,

$$U_{\text{eff}}(\delta) = 2 \times 3^{5/4}h^{-1/4}(\delta - \delta_c(h))^{3/2}. \qquad (5)$$

At high temperatures, the dissociation is driven by thermal activation over this barrier. Using the known theory describing the escape from such a potential well [1,2], we find the switching rate of a long Josephson junction from the superconducting state to the resistive state:

$$\Gamma_T(I) \propto \exp[-U_{\rm eff}/k_B T]. \quad (6)$$

Thus, at high temperatures, the standard deviation of the critical current σ increases with temperature and weakly depends on the magnetic field: $\sigma_T \propto T^{2/3}h^{1/6}$. Notice that σ_T increases with H, in contrast to the behavior of a small Josephson junction [1,2], where $\sigma_T \propto [I_c(H)]^{1/3}$ decreases with H.

We experimentally investigated the fluctuation-induced dissociation of the VAV pair by measuring the temperature and magnetic field dependence of the statistical distribution P of the switching currents $I < I_{c0}$ using techniques described in [6,10,19]. In Fig. 2(a), the temperature dependence of the switching current distribution measured at $H = 0$ is shown. At high temperatures the $P(I)$ distribution is temperature dependent; at low temperatures a saturation is observed. In Fig. 2(b), the standard deviation σ of $P(I)$ is plotted versus bath temperature T for two values of magnetic field. σ is well approximated by $T^{2/3}$ dependence on the temperature, and the standard deviation is larger for the higher field as predicted in the above analysis. As clearly seen in Fig. 2(b), σ decreases with temperature and saturates below a crossover temperature of $T^\star \approx 100$ mK. At $T < T^\star$, the dissociation of the VAV pair occurs through a macroscopic quantum tunneling process. The crossover temperature $T^\star \simeq \hbar\omega(\delta)/(2\pi k_B)$ is determined by the frequency $\omega(\delta)$ of small oscillations of the VAV pair. In the quantum regime, the frequency $\omega(\delta) = 3^{3/8}/\sqrt{\chi}[\delta - \delta_c(h)]^{1/4}$ determines the oscillatory energy levels $E_n \simeq \hbar\omega(\delta)(n + 1/2)$ of the pinned VAV state. These energy levels can be further studied experimentally by performing microwave spectroscopy as has been demonstrated for the case of a single Josephson vortex trapped in a long Josephson junction [15]. Such a measurement can be used to experimentally confirm [15] the measured crossover temperature.

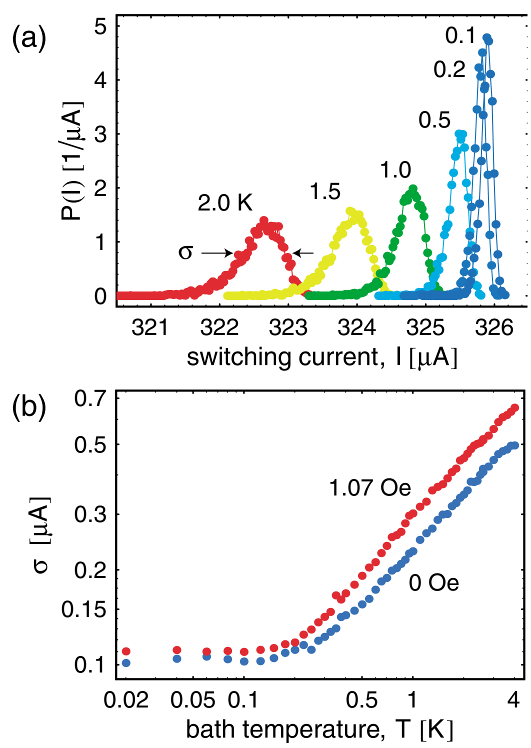

FIG. 2 (color online). (a) Switching current distributions $P(I)$ at $H = 0$ shown at different bath temperatures T. (b) Standard deviation σ of $P(I)$ distributions versus bath temperature for the two indicated values of magnetic field.

Neglecting dissipative effects, in the quantum regime the switching rate $\Gamma_Q(I)$ of the under-barrier dissociation can be estimated, as usual, in the WKB approximation, which yields

$$\Gamma_Q(I) \propto \exp\left(-\frac{36U_{\rm eff}(\delta)}{5\hbar\omega(\delta)}\right). \quad (7)$$

In this limit the standard deviation of the critical current σ is independent of temperature, and (similar to the high temperature case) it weakly increases with magnetic field.

The above analysis is based on an assumption that the pair's size is small with respect to the JJ length. We find that the Josephson phase escape in the form of the dissociation of the pair occurs (in normalized units) as $h \geq \frac{3}{4}(L/2)^{-4}$. In the opposite limit of very small magnetic field $h \lesssim (L/2)^{-4}$, the Josephson phase escape occurs homogeneously in the whole junction [20].

The analysis presented above is valid for small magnetic fields, $h \ll 1$. As the magnetic field h increases, the critical current $I_c(h)$ is suppressed, and only a *qualitative* description of the VAV pair dissociation can be carried out. In the general case, the pair size is $l_p \simeq (1 - (I/I_{c0})^2)^{-1/4}$ (instead of $l_p \simeq \delta^{-1/4}$ that is valid for $h \ll 1$), and the amplitude of the state is $\xi_p \simeq \arccos(I/I_{c0})$ (instead of $\xi_p \simeq \sqrt{\delta}$). Following a similar procedure as above [see Eqs. (3) and (4)], we obtain the standard deviation dominated by the thermal fluctuations:

$$\sigma_T \simeq \frac{T^{2/3}h^{2/3}}{\arccos(\frac{I_c(h)}{I_{c0}})}, \quad (8)$$

and in the quantum regime

$$\sigma_Q \simeq \frac{h}{(\arccos\frac{I_c(h)}{I_{c0}})^{8/5}}. \quad (9)$$

The standard deviation is determined by the magnetic field dependence of the critical current $I_c(h)$ that is given implicitly by the equation

$$h = \frac{3}{4}\sqrt{(1 - [I_c(h)/I_{c0})^2]}\arccos\left(\frac{I_c(h)}{I_{c0}}\right). \quad (10)$$

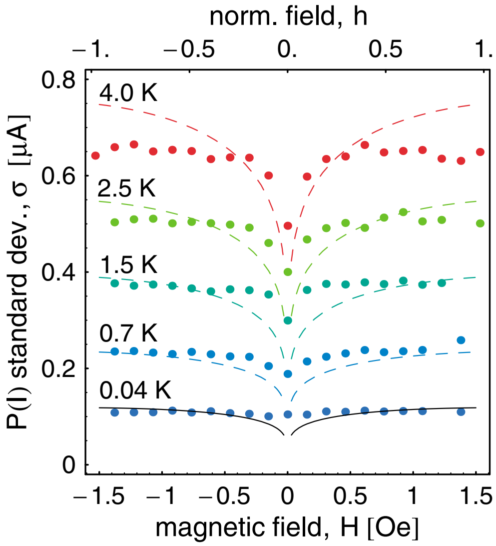

FIG. 3 (color online). Standard deviation σ of $P(I)$ distribution versus magnetic field H in the temperature range between 40 mK and 4.0 K: experiment (dots), theory [dashed lines, Eqs. (8) and (9)]. The region of magnetic field corresponds to the central lobe in the $I_c(H)$ dependence displayed in Fig. 1(d).

This dependence is shown in Fig. 1(d) by a dashed line. The discrepancy between analysis and numerics in the values of I_c is a consequence of the fact that the analysis has been carried out for junctions with length $L/2\pi \gg 1$ but the experimentally investigated system only barely meets that limit ($L \approx 10.5$). However, the analytical predictions and numerics are in good accord as the length of the Josephson junction is increased to $L \geq 20$ (data are not shown).

In Fig. 3, the measured field dependence of the switching current distribution width σ is shown for temperatures ranging from 40 mK to 4 K. The calculated dependencies $\sigma_{T(Q)}(h)$ are shown in the same figure by dashed lines. At each temperature, the distribution width σ has a minimum, pronounced in the thermal regime, at zero magnetic field. In qualitative agreement with the analysis given by Eqs. (8) and (9), σ shows an *increase* and a following *saturation* with magnetic field in both thermal and quantum regimes. This behavior is characteristic for the fluctuation-induced dissociation of a VAV pair.

In conclusion, we have shown that in the presence of a magnetic field the switching of a long Josephson junction from the superconducting state to the resistive one occurs through the nucleation and subsequent dissociation of a single vortex-antivortex pair. At low temperatures, we observe the pair dissociation by tunneling through the barrier.

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[20] We do not consider here very long Josephson junctions where even in the absence of magnetic field fluctuation-induced vortex-antivortex pairs are generated in the Josephson junction [11–13].