

# Quantum escape of the phase in a strongly driven Josephson junction

M. V. Fistul, A. Wallraff,\* and A. V. Ustinov

Physikalisches Institut III, Universität Erlangen-Nürnberg, D-91058 Erlangen, Germany

(Dated: July 28, 2003)

A quantum mechanical analysis of the Josephson phase escape in the presence of both dc and ac bias currents is presented. We find that the potential barrier for the escape of the phase is effectively suppressed as the resonant condition occurs, i.e. when the frequency  $\omega$  of the ac bias matches the Josephson junction energy level separation. This effect manifests itself by a pronounced drop in the dependence of the switching current  $I_s$  on the power  $W$  of the applied microwave radiation and by a peculiar double-peak structure in the switching current distribution  $P(I_s)$ . The developed theory is in a good accord with an experiment which we also report in this paper. The obtained features can be used to characterize certain aspects of the quantum-mechanical behavior of the Josephson phase, such as the energy level quantization, the Rabi frequency of coherent oscillations and the effect of damping.

PACS numbers: 05.45.-a, 74.50.+r, 03.75.Lm

Great attention has been devoted to the experimental and theoretical study of Josephson junctions in the presence of externally applied microwave radiation. The well known effects are Shapiro steps in the current-voltage characteristics, and features due to photon-assisted tunneling of quasiparticles [1]. Moreover, microwave radiation can be used to probe the *macroscopic quantum-mechanical* behavior of the Josephson phase such as the energy level quantization [2, 3, 4] and the recently observed quantum coherent Rabi oscillations [5, 6]. The observation of these effects has proven that the laws of quantum mechanics can be applied to dissipative systems. The interest in this field has also been boosted by the possibility to use Josephson junctions as superconducting qubits [5, 6].

The quantum mechanical properties of the Josephson phase can be studied most directly when the Josephson phase interacts resonantly with the external microwave radiation [2, 3, 5, 6, 7]. Such an interaction leads to the resonant absorption of photons, which induces transitions between the energy levels of the Josephson phase and thus to a pronounced increase of the population of excited levels.

It is also well known that thermal and quantum fluctuations lead to a premature *switching* of the junction from the superconducting state to the resistive state at a random value of dc bias  $I_s < I_c$ , where  $I_c$  is the critical current in the absence of fluctuations [1, 2, 3]. In this case, the escape of the Josephson phase can be characterized by the switching current distribution  $P(I_s)$  measured by ramping up the current at a constant rate. The resonant absorption of photons in a Josephson junction results in an *enhancement* of the rate of the phase escape at a particular dc bias  $I_r$  [2, 3, 4, 7]. The value  $I_r$  is found from the resonant condition as the frequency  $\omega$  (or  $n\omega$ , where  $n$  is an integer number, for multiphoton absorption [7]) matches the energy level separation

$$\hbar\omega = E_n(I_r) - E_m(I_r) . \quad (1)$$

Here,  $E_{n,m}(I_r)$  are the quantized energy levels of the Josephson phase in the presence of an externally applied dc bias  $I_r$ . The enhancement of the escape rate manifests itself by additional microwave-induced peaks in  $P(I_s)$ . This effect has been analyzed in detail in Refs. [2, 3, 4].

The crucial condition to observe such an enhancement of the escape rate is the small-signal limit, in which the microwave radiation only changes the population of individual levels in the well. To analyze this condition quantitatively we note that the fluctuation induced escape of the Josephson phase from excited levels has to be large. In the quantum regime the escape probability is governed by the exponent of the quantum tunneling rate which is proportional to the ratio of the barrier height to the attempt frequency

$$\frac{E_J(1-\gamma)^{3/2}}{\hbar\omega_p(1-\gamma)^{1/4}} ,$$

where  $E_J$  and  $\omega_p$  are the Josephson energy and the plasma frequency of the junction, and  $\gamma = I_s/I_c$ . To observe fluctuation-induced escape of the phase this exponent has to be of the order of unity, i.e.  $1 - \gamma \leq (\hbar\omega_p/E_J)^{4/5}$ . At the same time to observe the resonance the drive frequency  $\omega$  has to fulfill the resonance condition  $\omega = \omega_p [2(1-\gamma)]^{1/4}$ . Thus, an enhancement of the escape that can be directly interpreted as a microwave-induced increase of the excited level population, occurs approximately as

$$\left(\frac{\omega}{\omega_p}\right)^5 \leq \frac{\hbar\omega_p}{E_J} . \quad (2)$$

As we turn to a larger microwave frequency or to smaller value of the critical current density, the condition (2) is not valid any more, and a simple increase of the population of excited levels can *not* explain the resonantly induced escape of the Josephson phase. In this paper we show that in the limit  $(\omega/\omega_p)^5 \gg (\hbar\omega_p)/E_J$ ,

the presence of microwaves leads to an *effective suppression* of the potential barrier for the escape of the phase. This suppression is particularly strong when the resonance condition is satisfied. We show that this resonant effect does not only lead to a double peak structure of the  $P(I_s)$  distribution at the resonance but is also accompanied by a sharp drop in the dependence of the average switching current  $\langle I_s(W) \rangle$  on the power  $W$  of the applied microwave radiation. Note here, that the analysis of this effect based on the classical nonlinear dynamics for the Josephson phase has been presented in [8].

To analyze the problem quantitatively we write down the time-dependent potential energy of the Josephson phase  $\phi(t)$  in a small Josephson junction in the presence of both dc bias current  $I$  and the ac bias with amplitude  $\eta$  induced by microwaves:

$$U(\phi) = U_0(\phi) - E_J \frac{\eta}{I_c} \sin(\omega t) \phi, \quad (2)$$

$$U_0(\phi) = -E_J \left( \cos \phi + \frac{I}{I_c} \phi \right). \quad (3)$$

Here,  $U_0(\phi)$  is the potential energy of the Josephson junction in the absence of ac bias.

In analogy to the classical nonlinear problem [8], we write the coordinate  $\phi(t)$  as a sum of two terms: a quickly oscillating resonant term  $\xi(t)$  and a term  $\phi_0(t)$  slowly varying in time. Next, the quantum-mechanical average of  $\langle \xi(t) \rangle$  is calculated using perturbation theory with respect to the small amplitude  $\eta$ . Moreover, we truncate the dynamics of the Josephson phase in the potential  $U_0(\phi)$  to the two energy levels  $E_{n,m}$  which resonantly interact with the ac bias. We take into account the damping parameter  $\alpha$  that determines the strength of the resonant interaction. This damping can be governed by the quasi-particle resistance of the junction itself and/or dominated by the impedance of the bias leads at the frequency  $\omega$  [3, 6, 9, 10]. In linear approximation we obtain:

$$\langle \xi(t) \rangle = E_J \frac{\eta}{I_c} \text{Im} \left( \sum_{n,m} \frac{f_{nm}^2 e^{i\omega t}}{\hbar^{-1} E_{nm}(I) - \omega + i\alpha} \right), \quad (4)$$

where the energy level differences  $E_{nm}(I) = E_n(I) - E_m(I)$  and the matrix elements  $f_{nm} = \langle n | \hat{\phi} | m \rangle$  depend on the dc bias current. The values of  $E_n(I)$  and the corresponding eigenfunctions  $|n\rangle$  are found as solutions of the Schrödinger equation

$$-\frac{\omega_p^2}{2E_J} \Psi_n(\phi)'' + U_0(\phi) \Psi_n(\phi) = E \Psi_n(\phi). \quad (5)$$

Substituting the  $\langle \xi(t) \rangle$  term in the expression for the potential energy  $U(\phi)$  and taking the average over time

we obtain the effective potential energy  $U_{\text{eff}}(\phi_0)$  in the form

$$U_{\text{eff}}(\phi_0) = -E_J \left[ \frac{I}{I_c} \phi_0 + \cos \phi_0 \left( 1 - \frac{\eta^2}{2} \sum_{nm} \frac{f_{nm}^4}{(\hbar^{-1} E_{nm}(I) - \omega)^2 + \alpha^2} \right) \right]. \quad (6)$$

The dependence of the switching current  $I_s(W)$  on the microwave power  $W = k\eta^2/2$ , where  $k$  is the microwave coupling coefficient is obtained by making use of the condition that  $U_{\text{eff}}(\phi_0)$  has no extreme points. Thus, the shift in switching current  $\delta I_s(W) = (I_c - I_s(W))/I_c$  is determined by a solution of the transcendental equation

$$\delta I_s(W) = k^{-1} W \sum_{nm} \frac{f_{nm}^4}{(\hbar^{-1} E_{nm}(I_s) - \omega)^2 + \alpha^2}. \quad (7)$$

Note here, that as we derive Eq. (7) all fluctuation effects are neglected.

A good approximation of the dependence  $\delta I_s(W)$  can be obtained in the harmonic oscillator model. In this case, the term with  $(n = 0, m = 1)$  in right-hand part of the Eq. (7) is most important, and the energy level difference is  $E_{01} = \hbar\omega_p(2\delta I_s)^{1/4}$ . The typical calculated dependence of the switching current on the microwave power is presented in Fig. 1. The most peculiar feature of the dependence  $I_s(W)$  is a sharp drop as the microwave power is close to the critical value  $W_{cr}$ . The appearance of the critical value of microwave power has a simple physical meaning. As the microwave power  $W$  is less than  $W_{cr}$  the resonant interaction is a weak, and the ac induced suppression of the barrier is not enough to allow for the Josephson phase to escape. The  $I_s(W)$  curves also display a weak dependence on  $W$  in the limit  $W \leq W_{cr}$ , and this dependence becomes stronger in the limit of relatively high microwave power  $W \geq W_{cr}$ . Moreover, due to the multi-valued character of  $I_s(W)$  there is a particular region of microwave power where a switching current distribution  $P(I_s)$  with two peaks should be observed. Notice that, while the width of the peak corresponding to the larger value of the switching current  $I_s$  is due to the presence of fluctuations (thermal or quantum), the width of the lower peak is determined by the damping parameter  $\alpha$ . The magnitude of the critical current drop becomes smaller as the damping parameter  $\alpha$  increases, see Fig. 1.

The above analysis can be extended to include fluctuations. The presence of thermal fluctuations leads to a premature switching of the junction from the superconducting state with respect to the value  $I_s(W)$  obtained from Eq. (7). Taking into account thermal fluctuations we derive a transcendental equation that is similar to (7)

$$\langle \delta I_s(W) \rangle = \langle \delta I_s(0) \rangle +$$

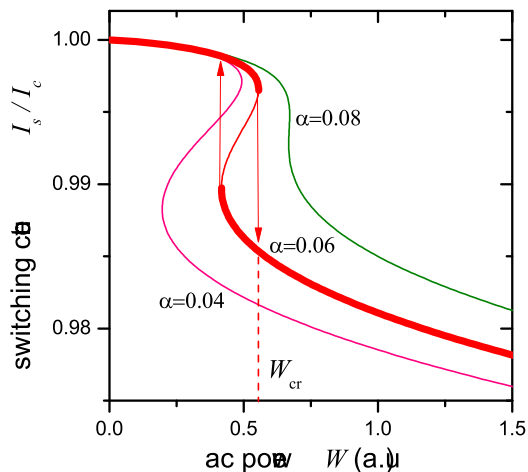


FIG. 1: Microwave power dependence of the switching current  $I_s(W)$  of a small Josephson junction. Dashed lines for  $\alpha = 0.06$  bound the region in which switching current distributions with two peaks should be observed. The frequency  $\omega$  of the ac drive is  $\frac{\omega}{\omega_p} = 0.4$  and the damping parameter is varied from  $\alpha = 0.04\omega_p$  to  $\alpha = 0.08\omega_p$ . These correspond to the junction's effective quality factor  $Q = \alpha^{-2}\omega^2$  ranging from 100 to 25.

$$k^{-1}W \sum_{nm} \frac{f_{nm}^4}{(\hbar^{-1}E_{nm}(\langle \delta I_s(W) \rangle) - \omega)^2 + \alpha^2}, \quad (8)$$

where  $\langle \delta I_s(0) \rangle$  is the fluctuation-induced shift of the switching current in the absence of ac bias. For temperatures above the crossover to the quantum regime  $\langle \delta I_s(0) \rangle = [\frac{k_B T}{2E_J} \ln(\frac{\omega_p I_c}{2\pi I})]^{2/3}$ , where  $\dot{I}$  is the bias current ramp rate [1]. By solving the Eq. (8) for  $\alpha = \text{const}$  we find that thermal fluctuations lead to smearing of the drop in the mean switching current and also shift the critical microwave power  $W_{cr}$  to smaller values, as illustrated in Fig. 2.

Equations (7) and (8) can also be used in the more complex case of a strongly non harmonic potential well  $U_0(\phi)$ . In such a regime, other terms in Eq. (7) become important. For example, by taking into account  $E_{02}$  transitions we should expect, according to Eq. (7), an additional jump (of much smaller value) in the dependence  $I_s(W)$  [11]. The microwave power  $W$  required to observe this jump is less than  $W_{cr}$ , the critical microwave power causing the main drop in the  $I_s(W)$  curve. The multiphoton absorption of the Josephson phase observed in Ref. 7, can be also be treated in the framework of this analysis. In this case Eqs. (2), (7) and (8) can still be used with the substitution of  $\omega$  by  $n\omega$ . Instead of the linear dependence of  $\langle \xi(t) \rangle$  on the amplitude  $\eta$ , one should expect the dependence  $\langle \xi(t) \rangle \sim J_n(\frac{\eta}{2e\omega})$ , where  $J_n$  is the Bessel function of order  $n$  [12].

The coupling coefficient  $k$  can be found by comparison of the observed  $\langle I_s(W) \rangle$  dependence with the one calcu-

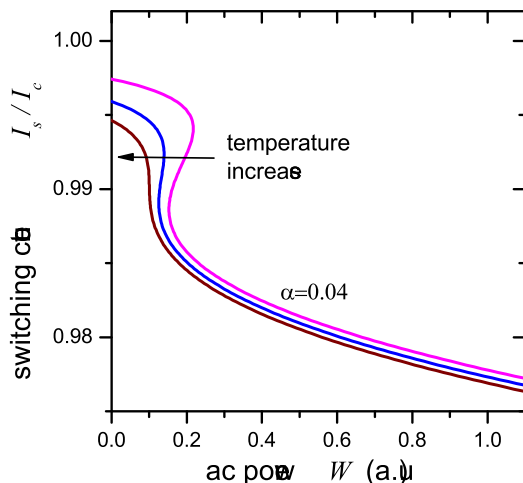


FIG. 2: Temperature dependence of the switching current of a small Josephson junction in the presence of ac bias at the frequency  $\omega = 0.4\omega_p$ . The reduction of the critical current due to thermal fluctuations is taken into account. The damping parameter is fixed  $\alpha = 0.04\omega_p$ . Temperatures shown are 0.1 K, 0.2 K and 0.3 K.

lated in Eq. (8). It allows to calculate the crucial parameter of coherent quantum-mechanical behavior, i.e. the microwave power dependent Rabi frequency of coherent oscillations as

$$\omega_R = \omega_p f_{nm} \sqrt{\frac{2W}{k}}. \quad (9)$$

It is worth to note that our analysis is also applicable to the problem of the microwave-induced escape of a pinned magnetic fluxon in an annular long Josephson junction, the dynamics of which can be mapped to the dynamic of a small Josephson junction [8, 13].

The analysis presented here was motivated by measurements of the switching current distribution  $P(I_s)$ , which we performed on a small ( $5 \times 5 \mu\text{m}^2$ ) Nb/AlO<sub>x</sub>/Nb Josephson tunnel junction. This junction had a fluctuation-free critical current of  $I_c = 278.45 \mu\text{A}$ , an effective capacitance of  $C = 1.61 \text{ pF}$  and a quality factor of  $Q \approx 45 \pm 5$  [7].  $P(I_s)$  distributions have been measured at a bias current ramp rate of 0.245 A/s at the temperature  $T = 100 \text{ mK}$ , which is well below the temperature  $T^* \approx 280 \text{ mK}$ , corresponding to the crossover between thermal escape and quantum tunneling for this sample. Microwaves in the frequency range between 10 GHz and 38 GHz have been applied to the sample and the microwave power was varied over a wide range. In Ref. 7 we have focused on the regime in which for small microwave powers the population of an excited state is enhanced and subsequent tunneling from the excited state is observed. In those experiments the drive frequency  $\omega$  has been chosen to satisfy the condition (2). Here,  $n$  is the number of photons involved in the transition. In this case the

$P(I_s)$  distributions were only weakly perturbed by the microwaves.

Here we present experiments for relatively large microwave frequencies  $n\omega/\omega_p$ . To observe a microwave-induced resonance under this condition, a larger microwave power has to be applied. In this case, the  $P(I_s)$  distributions are strongly perturbed by the microwaves. In Fig. 3a the dependence of the most probable switching current  $I_p$  on the microwave power  $W$  is plotted for a microwave frequency of  $\nu = \omega/(2\pi) = 24$  GHz. In Fig. 3b, the  $P(I_s)$  distributions are plotted for the three selected microwave powers indicated in Fig. 3a. It is observed that at small microwave powers the switching current distribution initially shifts to lower bias currents without developing a resonant double peak structure. At larger microwave powers – when the resonance condition is fulfilled – a double-peak distribution occurs. It is accompanied by a pronounced drop in critical current as the microwave power is further increased. The resonance presented here corresponds to  $n = 2$  photons with  $2\omega/\omega_p = 0.41$  [14].

The observed features in the  $P(I_s)$  distribution can not be explained by a simple enhancement of the Josephson phase escape due to a microwave induced increase of the population of an excited state [2, 4, 7]. We find a particular microwave power range in which two states (resonant and non resonant ones) coexist, indicating a microwave-induced bistability of the junction predicted by our theoretical analysis presented above. The characteristic behavior in the region of  $W \simeq W_{cr}$  is also in a good agreement with analysis above, compare Fig. 2 and Fig. 3a [15].

In conclusion we have carried out a quantum-mechanical analysis of the Josephson phase escape in a small Josephson junction in the presence of both dc bias and microwave currents. We find that in the absence of fluctuations and for large microwave frequency the dependence  $I_s(W)$  displays a pronounced drop at resonance and thereafter decreases smoothly. This behavior is explained by the effective suppression of the potential barrier by microwaves and is in a good accord with experiments carried out with small Josephson junctions. Similar behavior has been observed for a pinned Josephson vortex in a long annular Josephson junction [13].

We acknowledge the partial financial support of this project by the Deutsche Forschungsgemeinschaft (DFG) and by D-Wave Systems Inc.

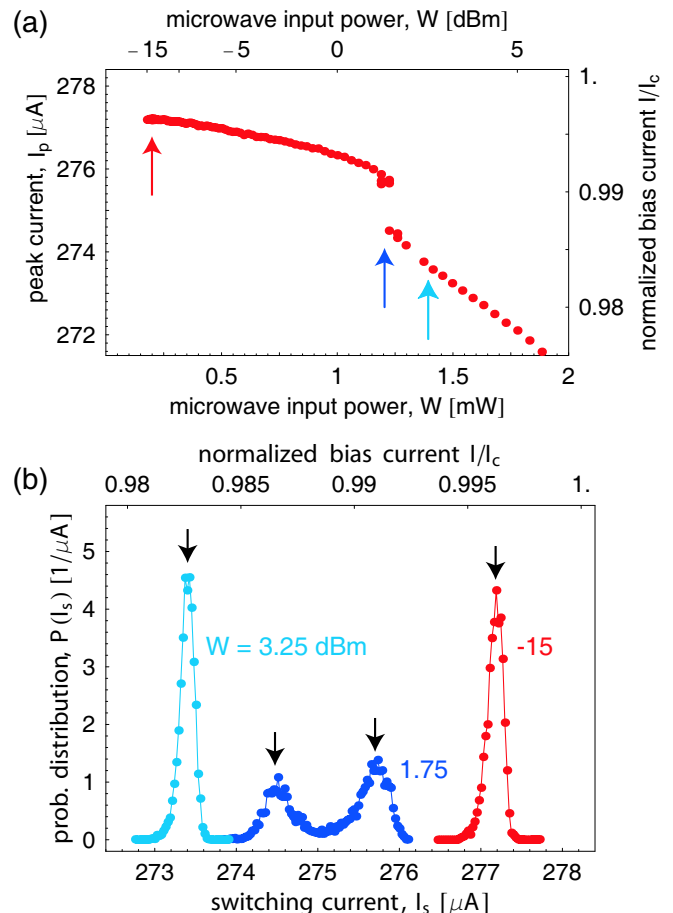


FIG. 3: (a) Peak switching current  $I_p$  versus microwave power  $W$  for a small Josephson junction. The applied microwave frequency was  $\nu = \omega/2\pi = 24$  GHz ( $n\omega/\omega_p = 0.41$ ). (b) Switching current distributions  $P(I_s)$  for the microwave powers indicated in (a). The peak switching currents  $I_p$  are indicated by arrows.)

- [3] J. M. Martinis, M. H. Devoret, and J. Clarke, Phys. Rev. B **35**, 4682 (1987)
- [4] A. I. Larkin and Yu. N. Ovchinnikov, J. Low. Temp. Phys., **63**, 317 (1986)
- [5] Yang Yu, Siyuan Han, Xi Chu, Shih-I Chu, and Zhen Wang, Science, **296**, 889 (2002).
- [6] J. M. Martinis, S. Nam, J. Aumentado, and C. Urbina Phys. Rev. Lett. **89**, 117901 (2002)
- [7] A. Wallraff, T. Duty, A. Lukashenko, and A. V. Ustinov, Phys. Rev. Lett. **90**, 037003, 2003.
- [8] M. V. Fistul and A. V. Ustinov, Phys. Rev. B, **63** 024508 (2000);
- [9] Yu. Makhlin, G. Schön, and A. Schnirman, Rev. Mod. Phys., **73**, 357 (2001).
- [10] M. V. Fistul, Phys. Stat. Sol., **233** 497 (2002).
- [11] The crucial condition to observe these additional jumps is the large values of matrix elements for ac bias induced transitions.
- [12] P.K. Tien and J.P. Gordon, Phys. Rev. **129**, 647 (1963).
- [13] A. Wallraff, J. Lisenfeld, A. Lukashenko, A. Kemp, M. V. Fistul, Y. Koval, and A. V. Ustinov, unpublished (2002).

\* Current address: Department of Applied Physics, Yale University, New Haven, CT 06520, USA

[1] M. Tinkham, *Introduction to Superconductivity*, 2nd edition, McGraw-Hill, New York (1996)

[2] Devoret M. H., J. M. Martinis and J. Clarke, Phys. Rev. Lett., **55** 1908 (1985).

- [14] In order to observe a similar behavior, i.e. a large drop in the dependence  $\langle I_s(W) \rangle$ , for a single-photon absorption process one would have to apply a larger microwave frequency  $\nu \simeq 50$  GHz, which is above the range of our microwave source.
- [15] Notice here that for a two-photon absorption process we have to use  $\omega = 2(2\pi\nu)$  and replace  $W$  by  $W^2$  in the Eq. (8).