Quantum dissociation of a vortex-antivortex pair in a long Josephson junction

M. V. Fistul[1], A. Wallraff[1,2], Y. Koval[1], A. Lukashenko[1], B. A. Malomed[3], and A. V. Ustinov[1]
[1] *Physikalisches Institut III, Universität Erlangen-Nürnberg, D-91058, Erlangen, Germany*
[2] *Department of Applied Physics, Yale University, New Haven, CT 06520, USA and*
[3] *Department of Interdisciplinary Science, Faculty of Engineering, Tel Aviv University, Tel-Aviv 69978, Israel*

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We report a theoretical analysis and experimental observation of the quantum dynamics of a single vortex-antivortex (VAV) pair confined in a long narrow annular Josephson junction. The switching of the junction from the superconducting state to the resistive state occurs via the dissociation of a pinned VAV pair. The pinning potential is controlled by external magnetic field H and dc bias current I. We predict a specific magnetic field dependence of the oscillatory energy levels of the pinned VAV state and the crossover to a *macroscopic quantum tunneling* mechanism of VAV dissociation at low temperatures. Our analysis explains the experimentally observed *increase* of the width of the switching current distribution $P(I)$ with H and the crossover to the quantum regime at the temperature of about 100 mK.

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Great attention has been devoted to the experimental and theoretical study of *macroscopic quantum phenomena* in diverse Josephson coupled systems [1, 2, 3, 4]. Most of these systems, e.g. dc biased single Josephson junctions (JJs), various SQUIDs and small Josephson junction arrays, contains a few *lumped* Josephson junctions, hence, they can be described by a few degrees of freedom (Josephson phases). At low temperature quantum-mechanical effects such as macroscopic tunneling, energy levels quantization [2, 3], and coherent oscillations [4] of the Josephson phase have been observed.

As we turn to *spatially extended* Josephson systems, including quasi-one-dimensional long Josephson junctions, parallel arrays and Josephson junction ladders, which present a particular case of interacting many particle systems, the analysis and observation of macroscopic quantum dynamics is more complex. These systems can support diverse nonlinear excitations, such as Josephson vortices (magnetic fluxons) and vortex-antivortex (VAV) pairs, which interact with inhomogeneities and linear (Josephson plasma) modes [5]. The classical dynamics of such excitations is well established, and in particular, the thermal fluctuation induced escape of a Josephson phase from the metastable state has been studied [6, 7]. It is necessary to stress that, while various macroscopic quantum-mechanical effects have been predicted [8, 9, 10, 11, 12, 13], only few of them were observed in experiments [14, 15]. In particular, studying the Josephson phase escape from the metastable state, the macroscopic quantum tunneling of a state with many vortices has been observed in [14], and recently tunneling of a single vortex and its energy level quantization have been measured [15].

After the quantum dynamics of a single vortex, the next step in the study of macroscopic quantum effects is the dynamics of a *single vortex-antivortex pair.* States containing many VAV pairs are relevant to thin superconducting films or large two-dimensional Josephson arrays close to the Kosterlitz-Thouless transition [1]. A *single* VAV pair naturally appears in a long *annular* JJ placed in the external magnetic field H parallel to the junction plane [16, 17] (Fig. 1a).

In this Letter, we report a theoretical analysis and experimental observation of quantum-mechanical effects in the dynamics of a single vortex-antivortex pair. We show that in the presence of an externally applied magnetic field H a single VAV pair can be nucleated in a long JJ. A pair is confined in a potential well created by externally applied magnetic field and dc bias current. Fluctuations, thermal and quantum, induce the internal oscillations of the pair. The switching of the JJ from the superconducting state to the resistive state occurs by *dissociation* of a pinned pair. At high temperature the dissociation takes place in a form of thermal activation over the barrier. At low temperature there occurs *macroscopic quantum tunneling* through the barrier.

First, we quantitatively analyze the penetration and following dissociation of a VAV pair in the presence of a small magnetic field H and a large dc bias, namely $\delta = \frac{I_{c0}-I}{I_{c0}} \ll 1$, where I_{c0} is the critical current of a long JJ in for $H=0$. In this case the Josephson phase is written as $\varphi(x,t) = \frac{\pi}{2} + \xi(x,t)$, where a small variable part ($\xi(x,t) \ll 1$) satisfies the equation:

$$\xi_{tt} - \xi_{xx} - \frac{\xi^2}{2} = -\delta - h\cos\left(\frac{2\pi x}{L}\right), \qquad (1)$$

where L is the junction length, and $h \propto H$ is the normalized external magnetic field. Here, coordinate x and time t are normalized to the Josephson penetration length λ_J and the inverse plasma frequency ω_p^{-1}, respectively. A particular solution of this equation satisfying the bound-

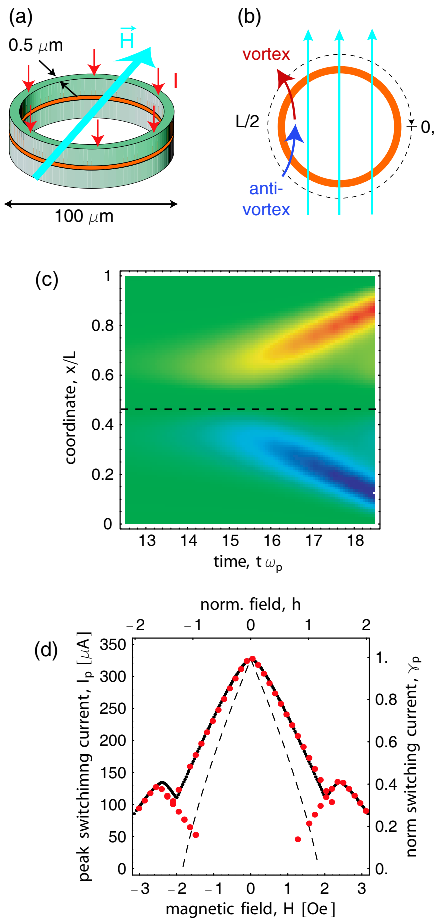

FIG. 1: (a) A schematic view of the system. A long annular Josephson junction *without any trapped vortices* is placed in the in-plane external magnetic field H and biased by uniform current I. (b) Generation of a confined vortex-antivortex pair with the center at $x = L/2$. (c) Numerically simulated evolution of the magnetic field distribution, as the Josephson junction switches into the resistive state. The emerging vortex and antivortex moving in opposite directions are seen. (d) The magnetic field dependence of the critical current for the annular junction with $L \equiv \pi d/\lambda_J = 10.5$: experimental data measured at $T = 850$ mK (circles), numerically calculated $I_c(H)$ (dotted line), and $I_c(H)$ predicted by Eq. (11) (dashed line).

ary conditions $\xi_x = 0$ at $x = \pm\infty$ is written in the form

$$\xi(|x-x_1|,A) = \sqrt{2\delta}\left[\frac{3}{\cosh^2\left(\frac{|x-x_1|+A}{2}(2\delta)^{1/4}\right)} - 1\right] . \qquad (2)$$

Here, $x_1(t)$ is the center of a confined pair, and the parameter $A(t)$ which will be allowed to vary in time, determines the distance between vortex and antivortex.

The magnetic field and dc bias create a pinning potential for such a state. Assuming that the JJ length L is much larger than the size of the pair (which is $\simeq\ \delta^{-1/4}$), we substitute (2) in the Hamiltonian of the underlying sin-Gordon model and minimize it with respect to x_1, which readily amounts to setting $x_1 = L/2$. Then, we find an effective energy of the JJ as a function of A ($\dot{A}$ stands for the time derivative),

$$E(A) = \frac{m_{eff}(A)}{2}\dot{A}^2 + U_{pot}(A) \ ,$$

$$U_{pot}(A) \ = \ m_{eff}(A) - 12(2\delta)^{1/4}h\tanh\left(\frac{A(2\delta)^{1/4}}{2}\right) \ , \qquad (3)$$

where the effective mass of the VAV pair is

$$m_{eff}(A) \ = \ 18(2\delta)^{3/2}\int_{-\infty}^{A} dx\frac{\sinh^2\left(\frac{x(2\delta)^{1/4}}{2}\right)}{\cosh^6\left(\frac{x(2\delta)^{1/4}}{2}\right)} \ . \qquad (4)$$

The critical current in the presence of an external magnetic field is found by minimization of the energy $E(A)$ in A. Then, the critical current is found as the bias current at which the static solution disappears, which assumes that (in the absence of fluctuations) the confined pair dissociates into a set of free vortex and an antivortex moving in opposite directions. After some algebra, the critical current is found to be $\delta_c(h) \ = 2h/3$, and the corresponding critical value A_0 of A being determined by the condition $\sinh\left((2\delta)^{1/4}A_0/2\right) = 1$. Note that the critical current decreases linearly with the magnetic field, in contrast to the case of *linear* long JJs, where similar consideration yields $\delta_c^{\mathrm{lin}}(h) \ \propto \ h^{4/3}$. Both results are valid for the ideal uniform bias-current distribution and $L/2\pi \gg 1$.

Next, we performed direct numerical simulations for the finite length JJ. The numerically found magnetic field dependence of the critical current is shown in Fig. 1d (dotted line). The simulations clearly show the nucleation and subsequent dissociation of the VAV pair (see, Fig. 1c). However, there is a discrepancy between the analysis and numerics in the values of I_c (see, Fig. 1d). Analysis and numerics are in a good accord as the length of a Josephson junction is increased, $L \ \geq \ 20$. The lower branch of the critical current (see Fig. 1d (circles)) corresponds to the penetration of a well separated vortex and antivortex in the junction ($A \ \simeq \ L/2$). The quantum dynamics of such a state will be discussed elsewhere.

In the presence of thermal or quantum fluctuations the dissociation of the pinned VAV pair occurs at a random value of dc bias, i.e. at $\delta \ \geq \ \delta_c(h)$. Assuming weak fluctuations, $\delta - \delta_c(h) \ \ll \ \delta_c(h)$, we expand the energy of the state around $A = A_0$ ($\delta A \ = \ A - A_0$) as

$$E(A) \ = \ \frac{\chi h^{5/4}(\delta\dot{A})^2}{2} + \frac{3^{3/2}\sqrt{h}}{2}\left(\delta - \delta_c(h)\right)(\delta A) - \frac{h^2}{6}(\delta A)^3 \ , \qquad (5)$$

where χ is the numerical coefficient of order one. Thus, the problem of fluctuation induced dissociation of a confined VAV pair is mapped to a well known problem of particles escape from a cubic potential. The probability of the dissociation depends on the height of the effective potential barrier

$$U_{eff}(\delta) \;=\; 2\;\cdot 3^{5/4}h^{-1/4}(\delta-\delta_c(h))^{3/2} \quad . \qquad (6)$$

At high temperatures, the dissociation is driven by thermal activation over this barrier. Using the known theory describing the particle escape from such a potential well [1, 2], we find the switching rate of a long Josephson junction from the superconducting state to the resistive state, $\Gamma_T(I)$,

$$\Gamma_T(I) \;\propto\; \exp\left[-\frac{2\;\cdot 3^{5/4}h^{-1/4}\,(\delta-\delta_c(h))^{3/2}}{k_B T}\right]. \qquad (7)$$

Thus, at high temperature the standard deviation of the critical current σ should increase with temperature and weakly depends on the magnetic field: $\sigma_T \;\propto\; T^{2/3}h^{1/6}$. Notice that σ_T *increases* with H, in contrast to the behavior of a small Josephson junction where $\sigma_T \;\propto\; (I_c(H))^{1/3}$ decreases with H.

At low temperatures the dissociation of the VAV pair occurs through a *macroscopic quantum tunneling* process. Such a quantum regime is realized at $T < T_{cr}$, where the crossover temperature T_{cr} is determined by the frequency $\omega(\delta)$ of small oscillations of VAV pair, $T_{cr} \;\simeq\; \frac{\hbar\omega(\delta)}{2\pi k_B}$. In the quantum regime the frequency $\omega(\delta) \;=\; \frac{3^{3/8}}{\sqrt{\chi}}\,(\delta-\delta_c(h))^{1/4}$ determines the oscillatory energy levels E_n $(n \;=\; 0,1,2...)$ of the pinned VAV state, $E_n \;\simeq\; \hbar\omega(\delta)(n+1/2)$.

Neglecting the dissipative effects, in the quantum regime the switching rate $\Gamma_Q(I)$ of the under-barrier dissociation can be estimated, as usual, in the WKB approximation, which yields

$$\Gamma_Q(I) \propto e^{-\frac{36U_{eff}(\delta)}{5\hbar\omega(\delta)}} \;=\; \exp\left[\frac{36\sqrt{\chi}h^{-1/4}\,(\delta-\delta_c(h))^{5/4}}{5\;\cdot 3^{3/8}\hbar}\right] . \qquad (8)$$

In this limit the standard deviation of the critical current σ is independent of temperature and, (similar to the high temperature case), it weakly increases with magnetic field: $\sigma_Q \;\propto\; h^{1/5}$.

Notice, that the process described above of the penetration and subsequent dissociation of the VAV pair is based on an assumption that the pair size is small with respect to the JJ length. We find that the Josephson phase escape in the form of the dissociation of the pair occurs (in normalized units) as $h \;\geq\; \frac{3}{4}(L/2)^{-4}$. In the opposite limit of very small magnetic field $h \lesssim (L/2)^{-4}$ the Josephson phase escape occurs homogeneously in a whole junction [18].

The analysis presented above is valid for small magnetic fields, $h \;\ll\; 1$. As the magnetic field h increases, the critical current $I_c(h)$ is suppressed, and only a *qualitative* description of the VAV pair dissociation can be carried out. In the general case the pair size is $l_b \;\simeq\; (1-(I/I_{c0})^2)^{-1/4}$ (instead of $l_b \;\simeq\; \delta^{-1/4}$ that is valid for $h \;\ll\; 1$), and the amplitude of the state is $\xi_b \;\simeq\; \arccos(I/I_{c0})$ (instead of $\xi_b \;\simeq\; \sqrt{\delta}$). Following a similar procedure as above (see, Eqs. (3) and (4)) we obtain the standard deviation dominated by thermal fluctuations

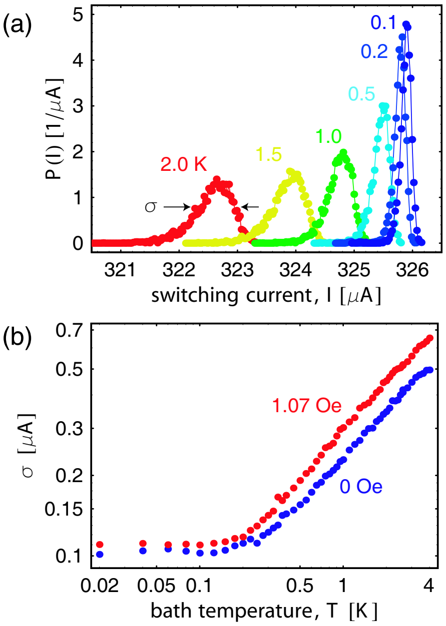

FIG. 2: (a) Switching current distributions $P(I-\langle I\rangle)$ at $I_H = 35\,\mu$A shown at different bath temperatures T. The data are plotted relative to the mean value $\langle I\rangle$ of the switching current at each temperature. (b) Standard deviation σ of $P(I)$ distributions versus bath temperature. Different colors correspond to different values of magnetic field.

$$\sigma_T \;\simeq\; \frac{T^{2/3}h^{2/3}}{\arccos\left(\frac{I_c(h)}{I_{c0}}\right)} \quad , \qquad (9)$$

and in the quantum regime

$$\sigma_Q \;\simeq\; \frac{h}{\left(\arccos\frac{I_c(h)}{I_{c0}}\right)^{8/5}} \quad . \qquad (10)$$

The standard deviation is determined by the magnetic field dependence of the critical current $I_c(h)$ that is given implicitly by equation (this dependence is shown in Fig. 1d by a dashed line):

$$h \;=\; \frac{3}{4}\sqrt{(1-(I_c(h)/I_{c0})^2)}\arccos\left(\frac{I_c(h)}{I_{c0}}\right) \quad . \qquad (11)$$

A most important consequence of this analysis is the *saturation* of $\sigma_{T(Q)}$ at moderate magnetic fields. The calculated dependencies $\sigma_{T(Q)}(h)$ are shown in Fig. 3 by dashed lines.

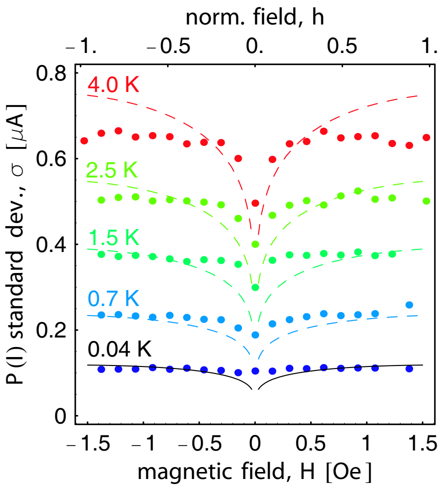

FIG. 3: Standard deviation σ of $P(I)$ distribution versus magnetic field H in the temperature range between 40 mK and 4.0 K: experiment (dots), theory (dashed line)s.

Next, we turn to an experimental study of Josephson phase escape in long annular JJs subject to an in-plane magnetic field. The junction of diameter $d = 100\,\mu$m and width $w = 0.5\,\mu$m was etched from a sputtered Nb/AlO$_x$/Nb thin film trilayer and patterned using electron-beam lithography [20]. Having a relatively small junction width w is essential for the observation of quantum effects, as the effective mass given by Eq. (4) (in natural units) is proportional to w. The critical current density of the junction was about 220 A/cm^2, which corresponds to $\lambda_J \approx 30\,\mu$m and $L \equiv \pi d/\lambda_J \approx 10.5$. The measurement of the critical current versus magnetic field shows that no Josephson vortices are trapped in the junction, see Fig. 1d. The magnetic field dependence of the critical current displays a linear decrease in a broad range of magnetic fields, which is consistent with the analysis presented above.

We have measured the temperature and magnetic field dependence of the switching current distribution $P(I)$, see Fig. 2. The details of measurements have been discussed elsewhere [6, 7, 10, 15]. As shown in Fig. 2, the width of distribution σ decreases with temperature and saturates below 100 mK. This behavior indicates that at low temperatures the VAV dissociation in the quantum regime is observed.

For each temperature, the distribution width σ has a minimum at zero magnetic field. Its magnetic field dependence displays a peculiar *increase* and a following *saturation* with magnetic field, see Fig. 3, both in thermal and quantum regimes. This behavior is characteristic for fluctuation induced dissociation of the VAV state. The $\sigma(h)$ dependence is in a qualitative agreement with the theoretical analysis given by Eqs. (9) and (10).

In conclusion, we have shown that in the presence of a magnetic field the switching of a long Josephson junction from the superconducting state to the resistive one occurs through the nucleation and subsequent dissociation of a single vortex-antivortex pair. At low temperatures we observe the pair dissociation, in the form of the tunneling under the barrier, in the direct experiment. The oscillatory energy levels of the pair can be further studied experimentally by applying an external resonant microwave radiation, similar to the case of a single Josephson vortex trapped in long Josephson junction [15, 19].

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