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Physica C 368 (2002) 324–327

PHYSICA C

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Testing a state preparation and read-out protocol for the vortex qubit

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Abstract

A double-well potential for a Josephson vortex can be formed by applying an external magnetic field in the plane of a heart-shaped long Josephson junction. At low enough temperatures, such a system is expected to behave as a quantum two-state system and hence is a candidate for a qubit. We have designed a model system of such a vortex qubit and experimentally demonstrate here a protocol for the preparation and read-out of its states in the classical regime. The measurement results agree well with the theoretical analysis of the vortex dynamics. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Josephson junction; Josephson vortex; Fluxon; Macroscopic quantum coherence; Qubit; Quantum computing

1. Introduction

Circuits of Josephson junctions have recently been shown to be possible good candidates for quantum computation [1–3]. An important issue in the operation of such superconducting solid state qubits is the preparation of the initial state of a qubit and the read-out of its final state after a desired quantum manipulation has been performed. We have proposed using a vortex trapped in a heart-shaped long Josephson junction as a *vortex qubit* [4]. The vortex in such a system can be viewed as a particle with a magnetic dipole moment, which is directed perpendicular to the junction, interacting with an external field [5]. The magnetic dipole interaction gives rise to an effective double-well potential for the vortex [4]. For small junction width [6], at ultra-low temperatures the dynamics of the vortex in such a potential is predicted to be quantum [7,8]. In the classical limit, the two basis states of the vortex qubit correspond to the vortex localized in the left or right well of the potential. Throughout this paper these states are labeled (1) and (2), respectively.

In the quantum regime, which is defined by the characteristic energy scale discussed below, the coupling between the two states depends exponentially on the size of the energy barrier separating them. The energy barrier can be tuned in a wide range by changing the magnetic field applied to the junction. At low fields, the vortex tunnels through the barrier, and thus coupling between the two states appears. At high fields, however, tunneling is essentially suppressed and the vortex remains localized in one of the states. Thus, by applying a sufficiently large field the system can be

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PII: S0921-4534(01)01190-X

switched into the classical regime in which the quantum states of the vortex are coupled exponentially weak.

Here we propose a protocol for the preparation and read-out of the vortex qubit states in the classical regime and test it experimentally. We are able to manipulate the vortex states by varying the magnetic field amplitude and its direction, and by applying a bias current to the junction.

The dynamics of a long Josephson junction is governed by the sine-Gordon equation (SGE) for the one-dimensional phase difference $\phi(q,t)$ between the superconducting wave functions in the two electrodes of the junction [9]. Here q denotes the coordinate along the junction normalized to the Josephson length, λ_J, and t, the time, normalized to the inverse plasma frequency ω_p^{-1}. The solitary wave solution of the unperturbed SGE

$$\phi^{v}(q,t) = 4\arctan\left(\exp\left(\frac{q-vt}{\sqrt{1-v^2}}\right)\right) \tag{1}$$

corresponds to a vortex of supercurrent, which carries a magnetic flux of a single flux quantum Φ_0, localized around vt and traveling with the velocity v. It is possible to treat the dynamics of a vortex in terms of a relativistic particle with a rest mass of $8E_0$, moving in one dimension. The characteristic energy scale is given by the Josephson energy $E_0 = \Phi_0 j_c \lambda_J w/2\pi$, where w denotes the width of the junction. This energy scale is proportional to the junction width. Since the expected quantum effects scale exponentially with the energy, a very narrow junction with $w \approx 0.3$ μm must be produced for the system to behave as a quantum system, with a vortex rest mass of approximately 10^{-3} of the electron mass. A technology for producing such narrow junctions was tested in Ref. [6]. Numerical calculations [10,11] show that for a width of $w \approx 0.3$ μm the tunnel rate can be tuned between kHz and GHz range. The junction used here for demonstrating the preparation and read-out procedure ($w = 3$ μm) was fabricated solely for the purpose of testing the measurement protocol.

The perturbed SGE [9]

$$\frac{\partial^2\phi}{\partial q^2} - \frac{\partial^2\phi}{\partial t^2} - \sin\phi = -\gamma + \kappa\frac{\partial(\vec{h}\vec{n})}{\partial q} + \alpha\frac{\partial\phi}{\partial t} - \beta\frac{\partial^3\phi}{\partial x^2\,\partial t} \tag{2}$$

takes into account the influence of the bias current density γ normalized to the critical current density j_c of the junction, the quasiparticle damping coefficient α, and the surface impedance losses β.

An external magnetic field with magnitude $h = |\vec{h}|$ and angle Θ is taken into account by the term $\kappa\partial(\vec{h}\vec{n})/\partial q$ in Eq. (2). The field is normalized by $H_0 = \Phi_0/\lambda_J d$, where d denotes the magnetic thickness of the junction. The vector $\vec{n}$ is the normal vector of the junction and κ is a coupling factor, determined by the specific geometry of the junction.

The field $\vec{h}$ interacts with the magnetic moment of the vortex, which is directed along $\vec{n}$. The exact shape of the interaction potential is determined by a convolution of the vortex magnetic profile with the field density profile induced by the external field [5]. Features of spatial extent χ smaller than λ_J are strongly diminished due to this convolution by a factor of approximately χ/λ_J. This regards variations of the shape as well as variations of the junction width. Microscopic inhomogeneities due to sample fabrication will influence the potential very weakly. Inhomogeneities on a larger scale can be compensated by the variation of the two magnetic field components.

By shaping the junction, a wide range of vortex potentials can be designed. In particular, the potential energy minima occur at locations, in which the vortex magnetic moment is aligned parallel to the field. In a heart-shaped junction two of these states exist, as shown in Fig. 2a for $\Theta = 0$, corresponding to a double-well potential for the vortex [4].

As long as the force exerted on the vortex by the bias current is smaller than the pinning force, the vortex remains confined to one of the potential minima. If the pinning force is exceeded by the driving force, the vortex starts to move. The vortex depinning current γ_{dep} depends on the magnitude and direction of the applied field. As the vortex gets depinned, a voltage jump from zero to a finite voltage is detected.

2. Preparation

The preparation procedure of the initial state of the vortex qubit is illustrated in Fig. 1. By applying

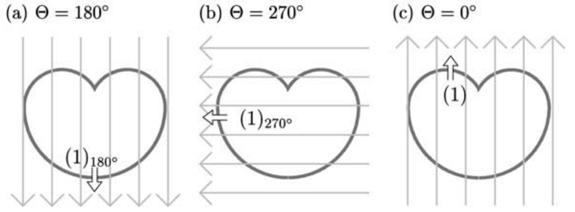

Fig. 1. Preparation of state (1). Top view of the heart-shaped junction. The field direction is indicated by large gray arrows, the vortex position is shown by the small white arrow.

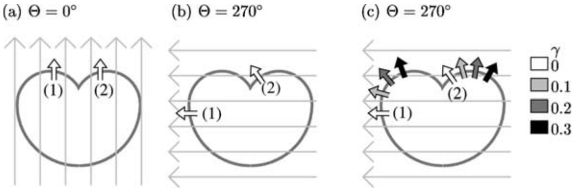

Fig. 2. Read-out of states. Small arrows denote the fluxon positions at zero bias. Long arrows denote the field direction.

the external magnetic field under an angle of $\Theta = 180°$ the vortex can be reliably trapped at the "bottom" of the heart, where it is aligned with the external field. Trapping is achieved by lowering the bias current from a value $\gamma > \gamma_{\text{dep}}$ to zero.

From the location indicated in Fig. 1a (the "bottom" of the heart), the vortex can be guided to one of the two stable positions at $\Theta = 0°$ by rotating the field either counterclockwise or clockwise. Fig. 1b shows the external magnetic field at an angle of $\Theta = 270°$, with the vortex position marked by $(1)_{270°}$. Finally, the field is rotated to an angle of $\Theta = 0°$, leading to state (1) as depicted in Fig. 1c. The preparation of state (2) simply requires a counterclockwise field rotation.

3. Read-out

The vortex state can be determined by a measurement of its depinning current γ_{dep}. If the forces pinning the vortex in the two minima are distinct, different depinning currents corresponding to the two different vortex states can be measured. This result is the read-out of the classical state of the vortex. The read-out in the quantum regime localizes the vortex in one of these states by increasing and rotating the magnetic field.

The distinction between the states is only possible for certain external field angles, at which the depinned vortex does not get retrapped in a neighboring potential minimum. The procedure depicted in Fig. 2 allows for a distinction of the vortex states. In Fig. 2a the both possible vortex positions for an external field applied under the angle $\Theta = 0°$ are shown. The vortex is located at either one of the two possible stable positions (1) or (2).

Rotating the field to an angle near $\Theta = 270°$, the vortex in either state moves to the corresponding positions marked in Fig. 2b. In Fig. 2c the shift of the vortex position due to an applied bias current is shown. The rest position of the vortex is driven by the bias current from the white filled arrows to the black arrows, which are the positions of the maximal pinning force exerted by the external magnetic field. Increasing the bias current any further depins the vortex and causes a voltage jump. [1]

4. Experimental results

We experimentally tested the protocol for the qubit preparation and read-out at 4.2 K. The heart-shaped junction used has a radius of 50 μm, a width of $w = 3$ μm and j_c close to 800 A/cm^2. The junction was fabricated using a standard Nb–AlO$_x$–Nb trilayer process [12]. A single vortex was trapped during cooling the junction through the critical temperature. Measurements of the depinning current were carried out at various angles of the external field. The magnetic field in the junction plane was applied using two mutually perpendicular coils. After retrapping the vortex at the external field angle of $\Theta = 180°$, as described above, the angle, at which the vortex state is read-out, is reached by either counterclockwise or clockwise rotation of the field.

The experimental results are compared to the numerical calculation in Fig. 3. Details on the

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[1] Due to the symmetry of the potential, the depinning currents of two states are equal at $\Theta = 270°$. By a small variation of Θ from this value the degeneracy can be lifted.

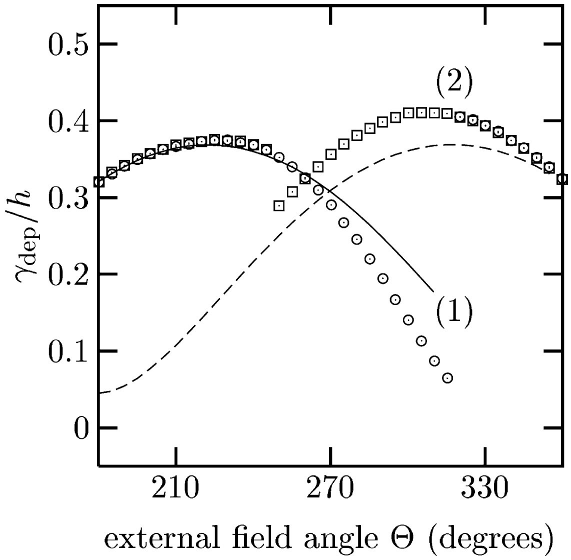

Fig. 3. The normalized depinning current c_{dep}/h plotted versus the angle Θ of the external field. Lines correspond to numerical calculation, squares—clockwise preparation of state (1) and circles—counterclockwise preparation of state (2) are the experimentally measured values.

calculation can be found elsewhere [10]. Since the depinning current γ_{dep} is expected to be proportional to the external field value h, we show the ratio γ_{dep}/h in dependence on the external field angle Θ. The lines correspond to the numerically calculated depinning currents of the states (1) and (2), while the points correspond to the measured values after the two different ways of preparation. It is clear that in the vicinity of $\Theta = 270°$ two distinctly different depinning currents correspond to the two different states of the vortex. Thus, by using the described read-out procedure we can determine the state of the vortex qubit.

5. Conclusion

We have designed and fabricated a classical vortex two-state system based on a heart-shaped long Josephson junction placed in an in-plane external magnetic field. We have proposed and successfully tested a protocol to reliably prepare and read-out the two vortex states. In particular, it is demonstrated that the vortex state can be determined using *a single-shot measurement* of the depinning current. The measured depinning currents have been compared to calculations based on perturbation theory and good agreement has been found. The developed method can be used to investigate thermal activation and quantum tunneling properties of vortices in very narrow long Josephson junctions.

Acknowledgements

We would like to thank Mikhail Fistul, Edward Goldobin and Timothy Duty for useful discussions and IPHT Jena for sample fabrication. This work was supported in part by Deutsche Forschungsgemeinschaft (DFG).

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