Radiation Spectrum of Stacked Josephson Flux–Flow Oscillators

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Abstract

The numerical investigation of the radiation emission by a system of two magnetically coupled long Josephson junctions is reported. Time dependent synchronized voltage response in the flux–flow regime is analyzed for the case of in–phase and out–of–phase oscillations in the junctions. Simulations show that Josephson junctions operating in the in–phase flux–flow mode may generate rf radiation power by a factor of more than four larger than that of a single Josephson junction. The radiation in the out–of–phase flux–flow mode is characterized by nearly completely suppressed amplitudes of odd harmonics and considerably damped even harmonics as compared to that of a single barrier junction. The dependence of the radiation power on the parameter spread between the junctions is investigated. The advantages of using stacked Josephson junctions as oscillators for the sub–mm wave band are discussed.

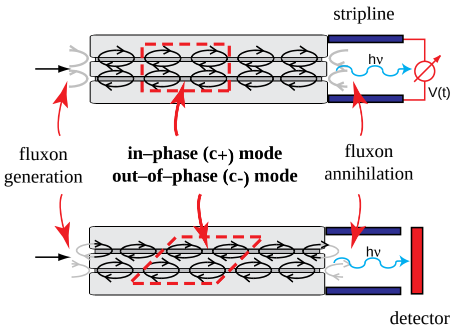
Modelling

Analysis of perfomance of a twofold stack of long Josephson junctions as a source of radiation:

- simulation of electromagnetic fields in the stack
- Fourier analysis of ac voltages at the edge of the stack facing the coupling circuit (eg stripline)

Two modes of oscillations in coupled long Josephson junctions:

- in–phase mode
- out–of–phase mode

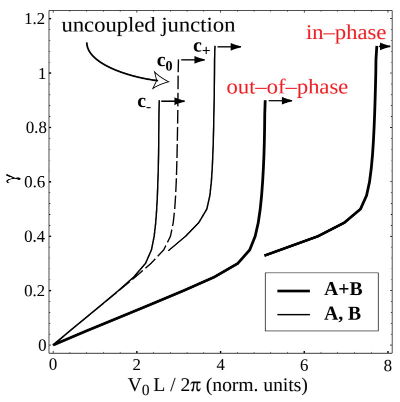


Simplifying assumptions:

- neglecting the loading effects
- annular geometry (to avoid complex interplay of flux–flow states with cavity resonances)

Current–voltage characteristics of an annular stack of long Josephson junctions

$$V = n \frac{h}{2e} \frac{2\pi}{L/\bar{c}} \qquad \bar{c}_{\pm} = \frac{\bar{c}}{\sqrt{1 \pm S}}$$



The junction parameters:

- $\alpha = 1.0$ • $\beta = 0$
- $L = 5$ • $N^A = N^B = 3$
- $S = -0.4$

were used unless others are explicitly stated.

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Theory

The theoretical model describing the dynamics of N–fold stacks of magnetically coupled long Josephson junctions was developed by Sakai, Bodin and Pedersen. In the present work we apply their model to twofold stacks and extend it including the case of LJJ's with arbitrary parameters. The normalized system of partial differential equations (PDE's) describing the dynamics of the stack is:

$$\begin{aligned} \frac{1}{1-S^2}\phi^A_{\tilde{x}\tilde{x}} &= \sin\phi^A + \phi^A_{\tilde{t}\tilde{t}} + \alpha^A\phi^A_{\tilde{t}} - \gamma + \sqrt{\frac{d'^A}{d'^B}}\frac{S}{1-S^2}\phi^B_{\tilde{x}\tilde{x}} \; ; \\ \frac{j_c^A}{j_c^B}\frac{d'^A}{d'^B}\frac{1}{1-S^2}\phi^B_{\tilde{x}\tilde{x}} &= \sin\phi^B + \frac{j_c^A}{j_c^B}\frac{C^B}{C^A}\phi^B_{\tilde{t}\tilde{t}} + \frac{j_c^A}{j_c^B}\frac{R^A}{R^B}\alpha^A\phi^B_{\tilde{t}} - \frac{j_c^A}{j_c^B}\gamma + \frac{j_c^A}{j_c^B}\sqrt{\frac{d'^A}{d'^B}}\frac{S}{1-S^2}\phi^A_{\tilde{x}\tilde{x}} \; , \end{aligned} \quad (1)$$

where

$$\tilde{t} = \omega_p^A t; \quad \tilde{x} = x/\lambda_J^A; \quad \gamma = \frac{j}{j_c^A}; \quad \alpha^{A|B} = \frac{\omega_p^{A|B}}{\omega_c^{A|B}} = \frac{1}{\sqrt{\beta_c^{A|B}}} = \frac{1}{R_N^{A|B}\sqrt{\frac{2\pi}{\Phi_0}j_c^{A|B}C^{A|B}}} \; . \quad (2)$$

The strength of the interaction scales with the coupling parameter S given by:

$$S = \frac{s_m}{\sqrt{d'^A d'^B}} = -\sqrt{\frac{\sinh\frac{d^A}{\lambda}}{\sinh\frac{d^A+d^m}{\lambda}}\frac{\sinh\frac{d^B}{\lambda}}{\sinh\frac{d^B+d^m}{\lambda}}} \; , \quad (3)$$

where $s_m = -\lambda/\sinh(d^m/\lambda)$ and the magnetic thickness

$$d'^{A|B} = \lambda\left(\coth\frac{d^{A|B}}{\lambda} + \coth\frac{d^m}{\lambda}\right) \; . \quad (4)$$

Harmonic Amplitudes

Periodic local voltage signals $V^A(t)$, $V^B(t)$ of the junctions can be expanded into Fourier series:

$$V^A(t) = \sum_{k=0}^{\infty} V_k^A \cos(\omega^A k t + \varphi_k^A) \; ; \quad (1)$$

$$V^B(t) = \sum_{k=0}^{\infty} V_k^B \cos(\omega^B k t + \varphi_k^B) \; . \quad (2)$$

The summed voltage for frequency locked junctions $\omega^A = \omega^B = \omega$ is

$$V^\Sigma(t) = \sum_{k=0}^{\infty} |V_k^\Sigma| \cos(\omega k t + \vartheta_k) \; , \quad (3)$$

where

$$|V_k^\Sigma|^2 = V_k^{A^2} + V_k^{B^2} + 2V_k^A V_k^B cos(\Delta\varphi_k) \; . \quad (4)$$

$|V_k^\Sigma|^2$ is the power the kth harmonic, depending on V_k^A and V_k^B and their phase difference $\Delta\varphi_k = \varphi_k^B - \varphi_k^A$. For stacks with and without parameter spread in the two phase locked modes $|V_k^\Sigma|$ is given by:

	out of phase	in phase
equal JJ $V_k^A = V_k^B$, $\delta\varphi_k = 0$	$\lvert V_k^\Sigma\rvert^2 = \begin{cases} 0 & , k = 1,3,5,\ldots \\ 4V_k^2 & , k = 2,4,6,\ldots \end{cases}$	$\lvert V_k^\Sigma\rvert^2 = 4V_k^{\Sigma^2}$, $k = 1,2,3,\ldots$
diff. JJ $V_k^A \neq V_k^B$, $\delta\varphi_k \neq 0$	$\lvert V_k^\Sigma\rvert^2 = V_k^{A^2} + V_k^{B^2} + \ldots$ $\ldots 2V_k^A V_k^B cos(k\pi + \delta\varphi_k)$	$\ldots 2V_k^A V_k^B cos(\delta\varphi_k)$

Numerics

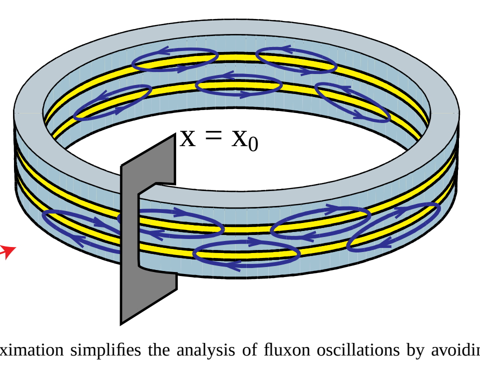
Solution to the Equations

With $R = \frac{R^A}{R^B}, J = \frac{j_c^A}{j_c^B}, C = \frac{C^B}{C^A}$ and $D' = \frac{d'^A}{d'^B}$ being the measures of the differences in junction parameters, the set of PDE's

$$\begin{aligned} \frac{1}{1-S^2}\phi^A_{\tilde{x}\tilde{x}} &= \sin\phi^A + \phi^A_{\tilde{t}\tilde{t}} + \alpha^A\phi^A_{\tilde{t}} - \gamma + \sqrt{D'}\frac{S}{1-S^2}\phi^B_{\tilde{x}\tilde{x}} \; ; \\ D'\frac{1}{1-S^2}\phi^B_{\tilde{x}\tilde{x}} &= \frac{1}{J}\sin\phi^B + C\phi^B_{\tilde{t}\tilde{t}} + R\alpha^A\phi^B_{\tilde{t}} - \gamma + \sqrt{D'}\frac{S}{1-S^2}\phi^A_{\tilde{x}\tilde{x}} \end{aligned} \quad (1)$$

was solved numerically according to the periodic boundary conditions

$$\begin{cases} \phi^{A|B}\big|_{\tilde{x}=0} = \phi^{A|B}\big|_{\tilde{x}=\ell} - 2\pi N^{A|B} \; ; \\ \phi^{A|B}_{\tilde{x}}\big|_{\tilde{x}=0} = \phi^{A|B}_{\tilde{x}}\big|_{\tilde{x}=\ell} \; . \end{cases} \quad (2)$$



This approximation simplifies the analysis of fluxon oscillations by avoiding the complicated interplay with cavity resonances. It is valid for very long junctions ($\ell\alpha > 1$) used in non–resonant practical oscillators.

Solutions were found by using explicit finite difference schemes up to fifth order. In each sequential point of the IV–curve the previous stationary state was used as initial condition. Stationary states on the resonances were reached after less than 100 normalized time units. Numerical stability was always achieved for $\Delta\tilde{x} = 0.02$ and $\Delta\tilde{t} = 0.005$.

Fourier Analysis of AC Voltages

Ac voltages were analyzed by simulating the time dependence of the local normalized voltages $V^{A|B}(t) \equiv \phi_t^{A|B}$ at an arbitrarily chosen point $x = x_0$ for annular geometry. Spectra were acquired for $0 < \gamma < 2$ at bias current steps of $\Delta\gamma = 0.05$.

Technical aspects of the FFT:

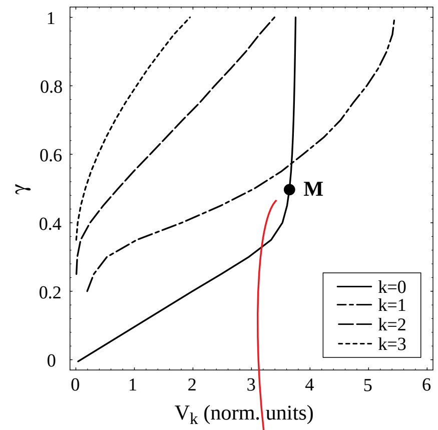
- 4096 data points for each FFT acquired at normalized rate $1/\delta\tilde{t} = 40$
- normalized Nyquits frequency $\tilde{\nu}_{max} = 20$
- peak broadening by Gaussian window function for accurate amplitude resolution

Single Uncoupled Junction

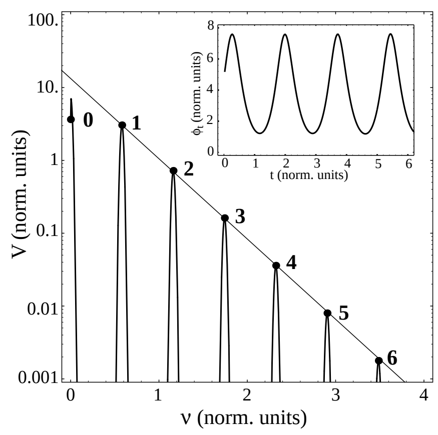
Normalized junction parameters

- $\alpha = 0.1$ • $\beta = 0$
- $L = 5$ • $N = 3$
- $S = 0$

DC component and first three harmonic amplitudes in dependence on the bias current



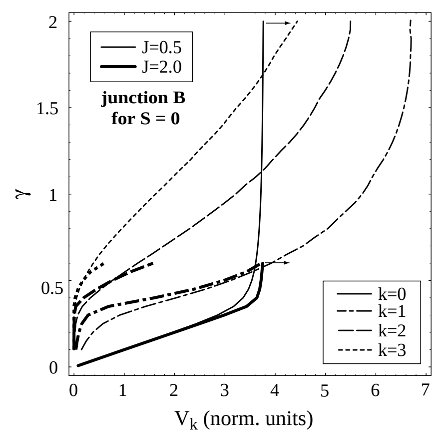
Spectrum and voltage profile at point M



Renormalized parameters

Different critical currents were simulated by analyzing the PDE for junction B with J≠1.

Variation of critical current: $J = j^A/j^B$



Variation of loss parameter α