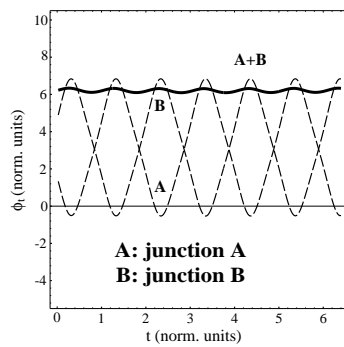


## Out-Of-Phase Mode

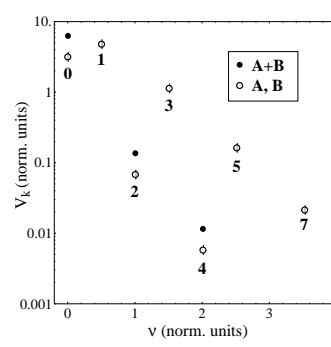
### Equal Junction Parameters

#### c. mode

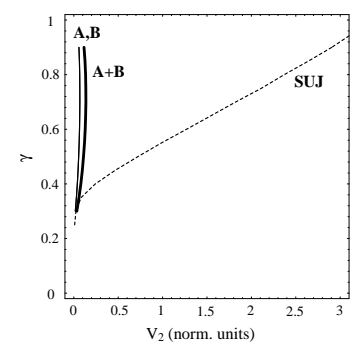
- suppression of even harmonics by magnetic images
- reduced frequency
- performance of single uncoupled junction is better than that of a stack in c. mode



The time dependent voltages  $V^A(x_0, t)$ ,  $V^B(x_0, t)$  and  $V^{\Sigma}(x_0, t)$  at the arbitrarily chosen coordinate  $x_0$  in a stack with equal junction parameters at  $\gamma = 0.5$ .



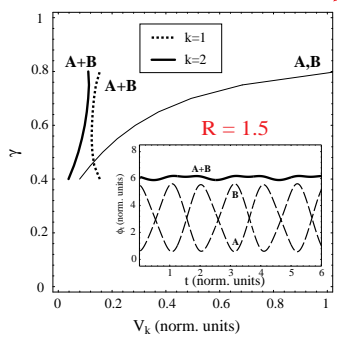
Spectrum of the voltage oscillations with harmonic amplitudes shown by open circles ( $V_k^{A+B}$ ) and small solid circles (total voltage  $V_k^{\Sigma}$ ) at  $\gamma = 0.7$ . Numbers indicate the harmonic index  $k$ .



The amplitudes of the second harmonic  $V_2^{\Sigma}$  vs. bias current  $\gamma$  of the stack of identical junctions synchronized in out-of-phase mode. The dashed line shows  $V_2^{\Sigma}$  of the single uncoupled junction (SUJ) for comparison.

### Spread in Junction Parameters

#### Spread in resistances: $R = R^A/R^B$



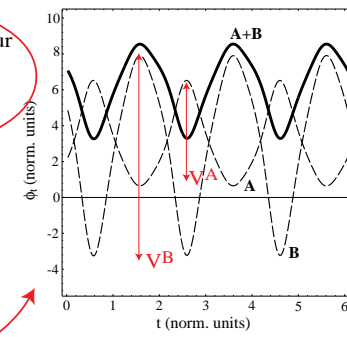
- phase shifts in harmonics occur
- amplitudes remain constant
- even harmonics almost cancel
- odd harmonics grow

also see frame on **Harmonic Amplitudes**

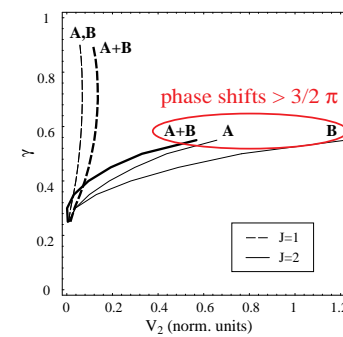
- amplitudes change  $V^A/V^B \approx 1/J$
- phase shift  $\delta\phi_k$  appears

Solid lines show  $V_2^{A,B}$  (thin line) and  $V_2^{\Sigma}$  (thick line) as a function of  $\gamma$ .  $V_1^{\Sigma}$  (dotted line) is plotted for reference. The voltage profiles  $V^A(x_0, t)$ ,  $V^B(x_0, t)$  and  $V^{\Sigma}(x_0, t)$  at  $\gamma = 0.5$  are shown in the inset.

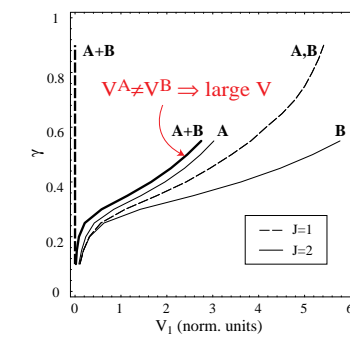
#### Spread in critical currents: $J = j^A/j^B \neq 1$



The evolution of  $V^{A,B}(x_0, t)$  and  $V^{\Sigma}(x_0, t)$  in time for  $J = j^A/j^B = 2$  at  $\gamma = 0.5$ .



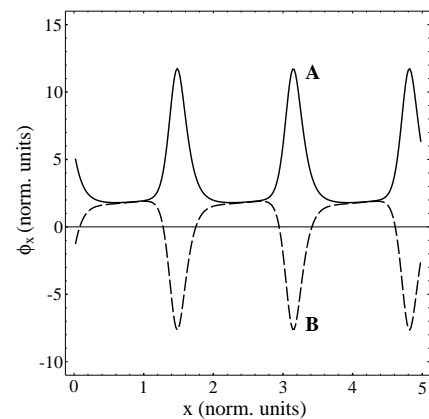
The second harmonic amplitudes  $V_2^{A,B}$  and  $V_2^{\Sigma}$  plotted vs.  $\gamma$  for  $J = 1$  (dashed lines) and  $J = 2$  (solid lines).



The first harmonic amplitudes  $V_1^{A,B}$  and  $V_1^{\Sigma}$  plotted vs.  $\gamma$  for  $J = 2$  (solid lines) and  $J = 1$  (dashed lines).

## Magnetic Images

### a) Out-of-phase, $N^A = 3, N^B = 0, S = -0.5$



The strong suppression of even harmonic amplitudes in the out-of-phase mode can be explained by considering the interaction of the junctions via the magnetic coupling. Fluxons moving in  $JJ^A$ , with magnetic field profile  $\bar{\phi}_x^A(x)$ , create the image profile  $\bar{\phi}_x^B(x)$  in  $JJ^B$ , as can be seen in plot a). The image profile in good approximation is given by

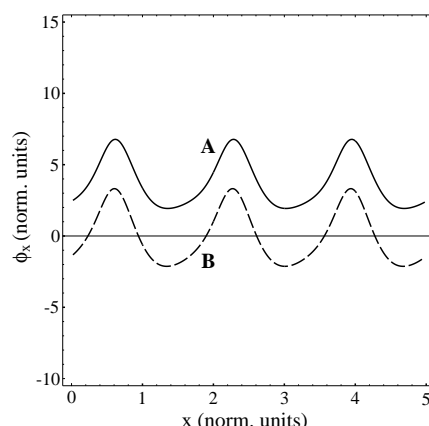
$$\bar{\phi}_x^B(x) \approx \text{const} - \epsilon \bar{\phi}_x^A(x), \quad (1)$$

where  $\epsilon$  is a constant close unity, for the values of  $S$  examined. Supposing existence of fluxons in  $JJ^B$  we get

$$\bar{\phi}_x^B(x) \approx \bar{\phi}_x^A(x) + \bar{\phi}_x^B(x) \approx \bar{\phi}_x^A(x) - \epsilon \bar{\phi}_x^A(x) + \text{const} \quad (2)$$

For the out-of-phase mode  $\bar{\phi}_x^B(x) = \bar{\phi}_x^A(x - T/2)$ . Fourier expansion of  $\bar{\phi}_x^B(x)$  shows that all odd (even) harmonics of  $\bar{\phi}_x^B(x)$  increase (decrease) by a factor of  $1 + \epsilon(1 - \epsilon)$ . Considering the addition of  $\bar{\phi}_x^B(x)$  and  $\bar{\phi}_x^A(x)$  with a phase shift of half a period results in vanishing odd and considerably reduced even harmonics as observed in the simulations of the out-of-phase mode.

### b) In-phase, $N^A = 3, N^B = 0, S = -0.5$

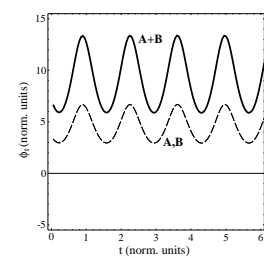


## In-Phase Mode

### Equal Junction Parameters

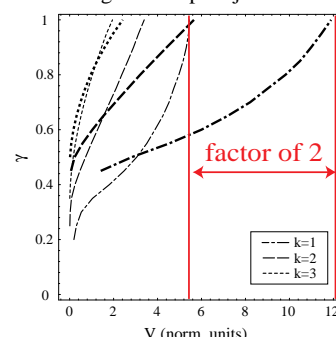
#### c+ mode

#### Local voltages at $\gamma = 0.5$



- gain in amplitude of  $V_1$  is larger than 2
- increase of frequency

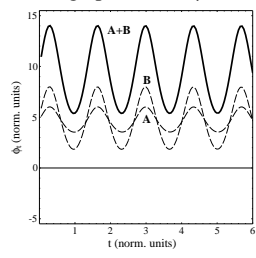
Harmonic amplitudes  $V_1, V_2, V_3$  of the stack (thick lines) and of the single uncoupled junction



### Spread in Junction Parameters

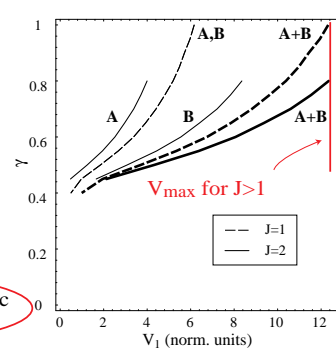
#### Spread in critical currents

#### Voltage profiles at $\gamma = 0.5$



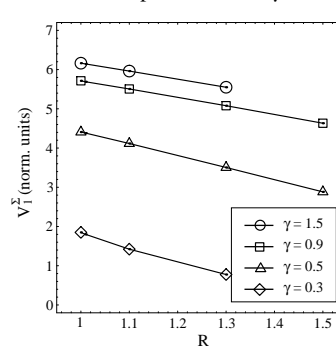
- almost constant first harmonic amplitudes for  $J > 1$

#### Harmonic amplitude $V_1$



#### Spread in loss parameters

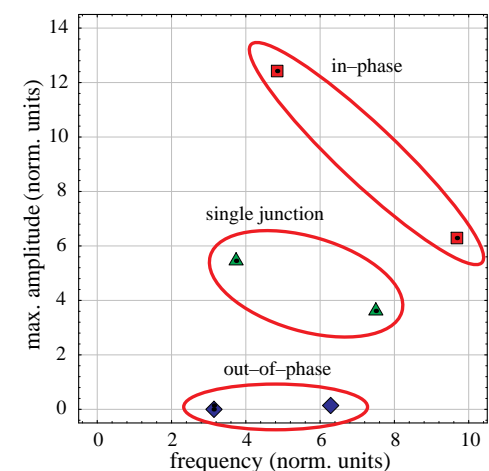
#### Harmonic amplitudes $V_1$ at $\gamma = \text{const}$ .



- linear decrease of  $V_1$  with  $R$

## Conclusions

- harmonic amplitudes are determined by magnetic images induced by the inductive coupling
- spread in junction parameters has strong influence on the harmonic amplitudes
- Feasibility of modes
  - In-phase
    - \* increase of power in low harmonics in comparison to single uncoupled junction
    - \* gain of more than a factor of 4 in the power of first harmonic
    - \* increased frequency
    - \* less stable with respect to parameter spread
  - Out-of-phase
    - \* less power in all harmonics than single uncoupled junction
    - \* reduced frequency



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